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# Assessment of Reinforced Concrete Half Joint According to EN -1992-1-1

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**Abstract.** A half-joint is a particular type of RC structure .When using precast beams in reinforced concrete frame structures, dapped beams ends are often required in the bearing area to keep the floor structure as thin as possible. The arrangement and detailing of the reinforcement is an important factor in the performance of a half-joint. For pre-tensioned members, the prestressing force and tendons should be ignored, but for post-tensioned members the prestressing force should be considered as an external force acting on the half-joint. This paper aims to calculate dapped beam ends according to the EN 1992-1-1.

**Keywords**: Half-joint, Reinforcement, Strut-and-tie model, Tensile force

## 1. Introduction

When using precast beams in reinforced concrete frame structures, dapped beams ends are often required in the bearing area to keep the floor structure as thin as possible. Because anchorage of the diagonal tie Zs is difficult due to the geometrical conditions in strut-and-tie models with an inclined reinforcement portion of 100 %, a combined strut-and-tie model comprising perpendicular and inclined suspension reinforcement is selected under normal conditions.
This paper aims to calculate dapped beam ends or a half-joint according to the EN 1992-1-1.[1]

## 2. Half-joint calculation in accordance with EN 1992-1-1

The calculation is based on a strut-and-tie model combined of perpendicular and inclined suspension reinforcement in accordance with DAfStb[1] Booklet 399.[2]

The equilibrium in the strut and-tie model is determined by iterative addition of the required stirrups and the recalculation of the centres of gravity of the reinforcement.
Strut-and-tie model consisting of inclined suspension reinforcement.

The inclined model is always used in combination with the model of perpendicular suspension reinforcement to prevent shearing of the half joint along the inclined bars. This model is more suitable for mapping the actual load-bearing behaviour of higher beam corbels.

[1] German Committee for Reinforced Concrete

## 2.1. Tensile force in the inclined reinforcement:

$F_{ZV} = (F_{ed} - F1_{ed}) \cdot$ inclined reinforcement portion **(1)**

## 2.2. Strut-and-tie model consisting of perpendicular suspension reinforcement

While the model of inclined suspension reinforcement results directly from the support geometry, the geometry of the model of perpendicular reinforcement is determined through the dimensioning of the nodes 1 and 2 of the inclined strut (D1), see Figure 2.

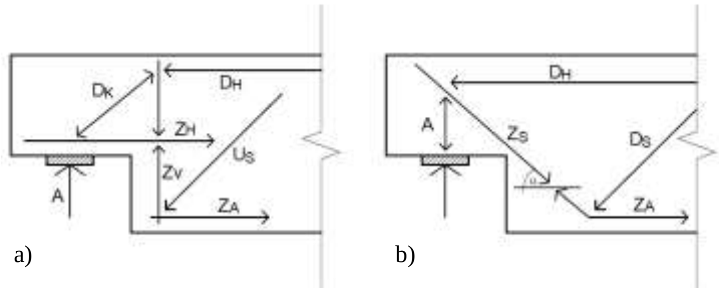


Fig. 1. Strut-and-tie model a) and an alternative Strut-and-tie model b)

It is assumed that the stress limit in node 2 (stress σcd2 depends on a4 ) is complied

$$\sigma Rdmax = k2 \cdot \nu' \cdot fcd \quad (2)$$

with; k2 and v' are assumed in accordance with the selected National Annex (NA).

$$\text{- for Germany: } k2 = 0.75,\ \nu' = 1.1 - fck / 500 <= 1.0 \quad (3)$$

$$\text{- for Austria: } k2 = 0.9,\ \nu' = 1.0 - fck / 250 <= 1.0 \quad (4)$$

The exact compression strut position with the dimensions of the nodes 1 and 2 is determined in the above expressions and the given border conditions such as the concrete cover and the centre of gravity of the suspension reinforcement and the horizontal reinforcement.By defining unfavourable half joint dimensions and/or if a high number of reinforcement layers is required, an inclination of the compression strut below 30° can result.

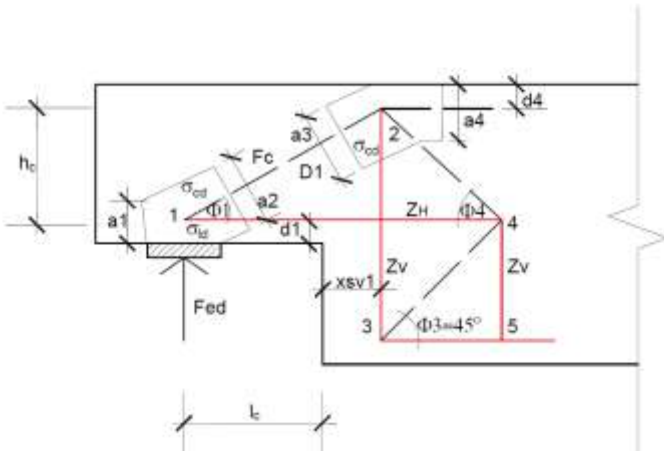


Fig. 2. The geometry of the strut-and-tie model

## 2.3. Tensile force in the vertical suspension reinforcement:

$$F_{ZV} = (F_{ed} - F1_{ed}) \cdot (1 - \text{ inclined reinforcement portion}) \quad (5)$$

Compression strut inclination:

$$\Phi_1 = \operatorname{atan}\left(\frac{h_C}{l_C}\right) \quad (6)$$

$$\sigma_{cd2} = \frac{F_{ZH}}{(b_k \cdot a4)} \quad (7)$$

$$b_k = \text{half-joint width; } F_{ZH} = F_{ZV} \cdot \frac{l_C}{h_C} \quad (8)$$

$$\text{Compressive strut force: } F_C = F_{ZV} / \sin(\Phi_1) \quad (9)$$

## 2.4. Horizontal forces due to the compressive strut geometry that are to be anchored:

$$F_{ZH} = (F_{ZV} + F1_{ed}) \cdot \left(\frac{l_C}{h_C}\right) + H_{ed} \quad (10)$$

## 2.5. Verification of the load-bearing capacity of the compressive concrete strut:

Compliance with the following condition must be verified

$$F_{ed} \le V_{rd,max} \quad (11)$$

$$V_{rd,max} = \alpha_{cv} \cdot b_v \cdot z \cdot v_1 \cdot f_{cd} / (\cot\theta + \tan\theta) \quad (12)$$

Where:
$\alpha_{cv}$ is a coefficient taking account of the state of the stress in the compression chord
$b_v$ is the minimum width between tension and compression chords
$z$ is the inner lever arm, for a member with constant depth , corresponding to the

bending moment in the element under consideration, the approximate value $z = 0.9 \cdot d$ may normally be used.
$v_1$ is a strength reduction factor for concrete cracked in shear
$f_{cd}$ is design value of concrete compressive strength
$\theta$ is the angle between the concrete compression strut and the beam axis perpendicular to the shear force, The angle $\theta$ should be limited

$$1 \leq \cot\theta \leq 2.5 \tag{13}$$

## 2.6. Verification of node 1 (bearing stress):

The verification of the compressive stress underneath the load plate is based on EN 1992-1-1: With the following conditions underneath the load plate:

$$\sigma_{ld} = \frac{F_{ed}}{(l_p \cdot b_p)} \leq \sigma_{rd} = k_2 \cdot \nu' \cdot f_{cd} \tag{14}$$

$k_2$ and $\nu'$ in accordance with the applicable National Annex (NA)
- for Germany: $k_2 = 0.75$, $\nu' = 1.1 - f_{ck}/500 \leq 1.0$
- for Austria: $k_2 = 0.9$, $\nu' = 1.0 - f_{ck}/250$

and in node 1:
$$a1(1) = 2 \cdot d_1 \tag{15}$$

$$a2(1) = \left(a1(1) \cdot \left(\frac{l_c}{h_c}\right) + l_p\right) \cdot \sin(\Phi_1) \tag{16}$$

$$\sigma_{cd} = \frac{F_c}{a2(1) \cdot b_p} \leq \sigma_{rd} = k_2 \cdot \nu' \cdot f_{cd} \tag{17}$$

$k_2$ and $\nu'$ as with node 1 for bearing stress, σld and σcd can be limited to $0.85 \cdot f_{cd1}$ in accordance with Schlaich/Schäfer.

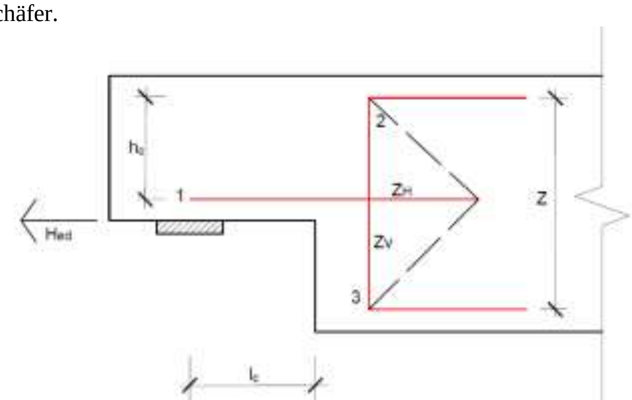


Fig. 3. Model for the back anchorage of the horizontal force

The tensile force of the vertical suspension reinforcement:

$$F_{ZV} = (F_{ed} - F1_{ed}) \cdot (1 - \text{inclined reinforcement portion}) + H_{ed} \cdot \frac{h_c}{z} \tag{18}$$

## 3. Worked example Design of Half Joint Using Strut-Tie Model

The beam in fig.4. is 7.0 m simply suported.The beam is 140cm x75 cm broad.The height of the half joint hk is 65 cm while the length of the half joint lk is 50 cm at each end.The beam supports a uniformly distributed load of 200 Kn/m.Assume C30/37 Mpa and B500B Mpa.[3]

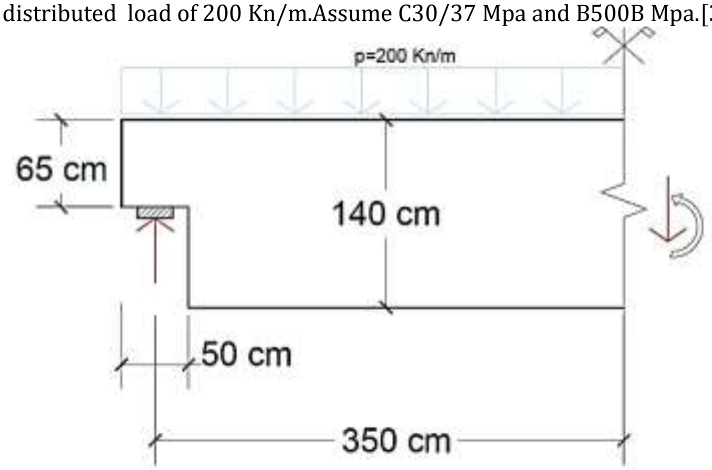


Fig. 4. A reinforced concrete "Half Joint" beam

Analysis:

A=p x (L/2+0.5/2)=200x3.75=750 Kn (19)

Bending moment at mid-span=750 x 3.50-200x3.75$^2$/2= 1218.75 Knm (20)

d1=c+ ϕ /2=4.0+2.0/2=5.00 cm (21)

d=h-d1=140-5.25=135cm (22)

CROSS SECTION

cross section values zu = 70cm Ac = 1.0500 m2 Ic =0.17150000 m4 (23)

BENDING DESIGN kd- method (x/d < 0.448), Myd = 1218.80 kNm (24)

ε1= -2.85 o/oo ε2s = 45.00 o/oo x/d = 0.07 z/d = 0.97 kd = 3.35 (25)

required Bottom longitudinal reinforcement (Asb) = 19.85 cm$^2$ μ = 0.19 % (26)

Provide:7R ϕ 20 =21.98cm$^2$ (27)

Top longitudinal reinforcement (As'): 4 ϕ 20 = 12.57 cm2 (28)

**Table 1.** Pulled up bearing by **EN 1992-1-1:2004 C 30/37 B500B**

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| | | | |
|---|---|---|---|
| support force | | F,ed = | 750.0 kN |
| horizontal force (UK) | | H,ed = | 150.0 kN |
| dist. axis of support - front edge notch | | e1 = | 25.0 cm |
| cross section of | b0 = 75.0 cm | h0 = | 140.0 cm |
| console | lk = 50.0 cm | hk = | 65.0 cm |

dimensions of sup bp = 36.0 cm ,lp =27.0 cm ,concrete cover c = 5.0 cm

distance upper and lower reinforce. (UE- and LE-centroid) do = 6.6 cm

**MODEL OF FRAMEWORK - GEOMETRY**

pressure strut D1 ( in console ) :dimensions l(hor) =56.0 cm h(vert) =43.2 cm
inclination $\phi_1$=37.63 degr.
node 1 (below) a1=18.0 cma2=32.9 cm
node 2 (above)a4=13.2 cmd4=6.6 cm
tied arch Zh ( horizontal reinforcement ) :
distance to upper edge beam h1 = 49.8 cm
tied arch Zv ( suspension reinforcement) :distance to front edge notch d1= 28.0 cm
distance from axis of support da = 53.0 cm
Tie ZV2 (suspension reinforcement 2):distance to front edge notch d2 =111.6 cm

**Internal forces**

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for vert. stirrups ZV1 (with contribution Zv = 819.1 kN
for vert. stirrups ZV2 Zv = 972.9 kN
horizontal tensile force Zh = 1175.7 kN

**Design**

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carrying capacity Fv=750.00 kN< V,rdmax = 1406.23 kN
trans. tens.force Ftd=86.25 kN
pressure at supp. $\sigma$,ld= 7.72N/mm2 <$\sigma$,rdmax=12.72 N/mm2
inpressure strut $\sigma$,cd=10.36N/mm2<$\sigma$,rdmax=12.72 N/mm2
suspens.stirr ZV1 Ac,req=18.84 cm2< Ac,exis =20.34 cm2
suspens.stirr.ZV2 Ac,req=22.38 cm2< Ac,exis =22.60 cm2
horizont.stirrups Ac,req=27.04 cm2< Ac,exis =31.40 cm2

**reinforcement**

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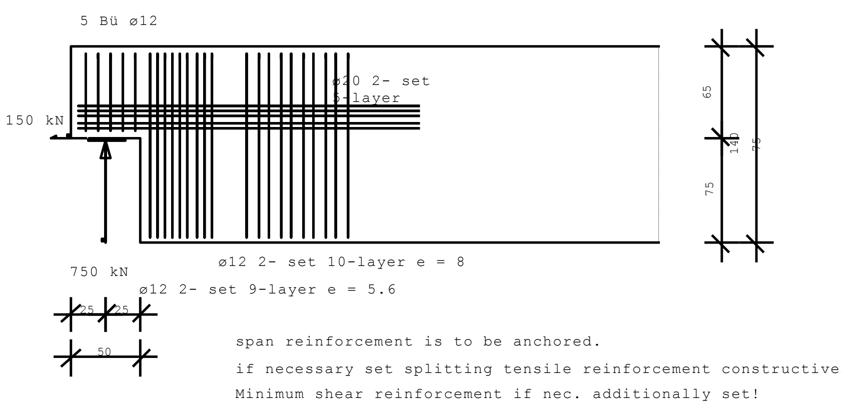


Fig. 5. A reinforcement of concrete “Half Joint” beam[4]

## Conclusions

The EC2 model avoid any unsafe predictions in the shear strength. Adequate anchorage should also be provided for the tensile steel reinforcement at the supports to prevent premature reinforcement slip failure.As shown in fig.5. the following stirrups are provided: ZV1 ø 12 2- set in 9 layers with 20.34 cm2, for ZV2 ø 12 2- set in 10 layers with 22.60 cm2 and for horizontal stirrups ø 20 2- set in 5 layers with 31.40 cm2 .The Eurocode 2 strut-and-tie model can be unsafe when av/d ratio is in between 1 and 2.

## References

1. EN 1992-1-1 (2004) (English): Eurocode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings [Authority: The European Union Per Regulation 305/2011,
2. Directive 98/34/EC, Directive 2004/18/EC]
3. Booklet 399 of the German Committee for Reinforced Concrete DAfStb)
4. Bhatt, P.,.Mac Ginlay, Th., Cho., B. S.: Reinforced Concrete Design to Eurocodes,(2014) Frilo Statik, Software for structural calculation and design