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AN OVERVIEW FOR CHAOS FRACTALS AND APPLICATIONS

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Abstract. The goal of this paper is to present, a summary of concepts from the Chaos and Fractals theory and some basic theory to understand how the Chaos and Fractal theory fundamentals work, for more to explain how is related the chaos theory with mathematics application. This paper will provide examples of how this new approach could be applied and how is chaos related to the Statistics. In this paper, I am going to introduce the concept of “fractal” dimension and some examples to calculate the dimensions. At the center of fractals is Mandelbrot set, otherwise is called the hard of the fractals, on this paper I am going to give a short description for that.

Keywords: Chaos, Fractals, Dimensions, Nonlinear, Dynamic, Logistic, Complex, etc.

1. The Chaos theory

The chaos theory is for some concepts that they offer an alternative that describes and explanations how is the behavior of some nonlinear systems (which are basically almost all naturally occurring physical, chemical, biological or social structures or systems). The name “Chaos” comes from the fact that nonlinear systems seem to behave chaotically or randomly from a traditional linear point of view. There are many natural systems whose behavior that can’t be described and explained by simply dividing the whole into its parts and study them separately from the rest of the system. For example, studying the behavior of an individual bee may not provide any insight into a beehive as a system because the bees colony’s behavior is driven by the cooperation and pheromone interaction between flowers. In a different example, of course the movement of water molecules in process of boiling the water might seem chaotic and random, but there are patterns of movements that change over time and tend to form similar structure.

Most natural systems change over time and this change does not happen in proportional and regular manner. A concept of proportional change is an idealization because real life phenomena change differently sometimes smoothly, sometimes not smoothly. The Chaos theory provides a theoretical framework and a set of tools for conceptualizing change and the changing system may have appeared to be chaotic from traditional (linear) perspective while it exhibits coherence, structure and patterns of motion from the global and nonlinear perspective.

Chaos is a fundamental property that possesses nonlinearity and it is very sensitive on initial conditions. Because of the nonlinearity in a chaotic system it becomes very difficult to make an exact or accurate predictions about the system over a given time interval. Weather forecasting is an example of how chaos theory effects the accuracy of predictions over a given time interval, but using the similar structured a meteorologist can predict how is going to move a hurricane. Through analyzing a weather pattern over time and different structures, meteorologists have been able to make better predictions of future weather based on this theory.

The dictionary definition of chaos is turmoil, unpredicted, turbulence, primordial abyss, and undesired randomness, but scientists will tell you that chaos is something extremely sensitive to

initial conditions. Chaos also refers to the question of whether or not it is possible to make good long-term predictions about how a system will act. A chaotic system can actually develop in a way that appears very smooth and ordered. Determinism is the belief that every action is the result of preceding actions. It began as a philosophical belief in Ancient Greece thousands of years ago and was introduced into science around 1500 A.D. with the idea that cause and effect rules. Newton was closely associated with the establishment of determinism in modern science. His laws were able to predict systems very accurately. They were deterministic at their core because they implied that everything that would occur would be based entirely on what happened right before. Henry Adams has described like this "Chaos often breeds life, when order breeds habit". Henri Poincaré was really the "Father of Chaos [Theory]". Chaos theory describes complex motion and the dynamics of sensitive systems. Chaotic systems are mathematically deterministic but is hard or impossible to predict. Chaos is more evident in long-term systems than in short-term systems. Behavior in chaotic systems is not periodic, meaning that no variable describing the state of the system undergoes a regular repetition of values. A chaotic system can actually develop gradually in a way that appears to be smooth and ordered, however. Chaos refers to the issue of whether or not it is possible to make accurate long-term predictions of any system if the initial conditions are known to an accurate degree. Chaos occurs when a system is very sensitive to initial conditions. Initial conditions are the values of measurements at a given starting time. The phenomenon of chaotic motion was considered a mathematical oddity at the time of its discovery, but now physicists know that it is very widespread and may even be the norm in the universe. The weather is an example of a chaotic system. In order to make long-term weather forecasts it would be necessary to take an infinite number of measurements, which would be impossible to do. Also, because the atmosphere is chaotic, tiny uncertainties would eventually overwhelm any calculations and defeat the accuracy of the forecast. The presence of chaotic systems in nature seems to place a limit on our ability to apply deterministic physical laws to predict motions with any degree of certainty.

One of the most interesting issues in the study of chaotic systems is whether or not the presence of chaos may actually produce ordered structures and patterns on a larger scale. It has been found that the presence of chaos may actually be necessary for larger scale physical patterns, such as mountains and galaxies, to arise. For centuries mathematicians and physicists have overlooked dynamical systems as being random and unpredictable. The only systems that could be understood in the past were those that were believed to be linear, but in actuality, we do not live in a linear world at all. In this world linearity is incredibly scarce. The reason physicists didn't know about and study chaos earlier is because the computer is our "telescope" when studying chaos, and they didn't have computers or anything that could carry out extremely complex calculations in minimal time. Below we can see that how are beavering the logistical functions $f(x)=kx(1-x)$, depending in values of $k= 1.5, 2.6$ and 3.3 and for certain value of initial points.

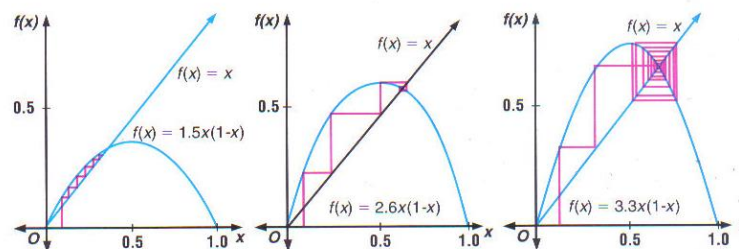


Figure 1

1.1. Chaos in the Real World

In the real world, there are three very good examples of unstable like the disease, political unrest, and family and community dysfunction. Disease is unstable because at any moment there could be an outbreak of illness in your body of some deadly disease for which there is no cure. This would cause some terror and chaos in your life. Political unrest is very unstable because people can revolt, throw over the government and create a wide war in different parts of the world. A war is another type of a chaotic system. A typical example of that is, the war in Middle East, which is hard to predict, how is going to be the future. Family and community dysfunction is also unstable because if you have a very tiny problem with a few people or a huge problem with many people, the outcome will be huge with many people involved and many people's lives with physical, economical and moral problems. Chaos is also found in systems as complex as electric circuits, measles outbreaks, lasers, clashing gears, heart rhythms, electrical brain activity, circadian rhythms, fluids, animal populations, and chemical reactions, and in systems as simple as the pendulum. It also has been all time present in stock market and sometimes it is possibly to occur a turmoil in the stock market.

2. Fractals

A fractal is a geometric shape that has symmetry of scale and self-similarity. This means that it is a shape that you could zoom in on a part of it an infinite number of times and it would still look the same. This is also called self-similarity. Computer-generated fractals are produced mathematically, and these can create detailed pictures of mountains, plants, waves, coastlines, and planets. Some mathematicians, such as Benoit Mandelbrot, study nature first, looking for fractal behavior.

Mathematics, Physics. a geometrical or physical structure having an irregular or fragmented shape at all scales of measurement between a greatest and smallest scale such that certain mathematical or physical properties of the structure, as the perimeter of a curve or the flow rate in a porous medium, behave as if the dimensions of the structure (fractal dimensions), are not only whole numbers (one, two or three) are greater than the spatial dimensions

Fractals are geometric shapes that are very complex and infinitely detailed. They are giving a procedure describing how to construct and defined a small section, where the small sections of them are similar to large ones. One way to think of fractals for a function $f(x)$ is to consider x , $g(x)$, $g(g(x))$, $g(g(g(x)))$, $g(g(g(g(x))))$, $g(g(g(g(g(x))))$), etc. Fractals are related to chaos because they are both complex systems that have similar properties.

3. Fractals and Benoit Mandelbrot set

Another technique to study the chaos is through the study of fractals. Fractals are sets that display, self-similarity at all levels of magnification, and they may have non-whole number dimension that is typical of chaotic attractors. Roughly speaking, self-similarity means that a set remains qualitatively similar in its spatial characteristics under contraction or magnification. Benoit Mandelbrot was a Poland-born French mathematician who greatly advanced fractals. When he was young, his father showed him the Julia set of fractals; he was not greatly interested in fractals at the time but in the 1970's, he became interested again and he greatly improved upon them, laying out the foundation for fractal geometry. He also advanced fractals by showing that fractals cannot be treated as whole-number dimensions; they must instead have fractional dimensions. Benoit Mandelbrot believed that fractals were found nearly everywhere in nature, at places such as coastlines, mountains, clouds, aggregates, and galaxy clusters.

Mandelbrot (1967, 1977, 1982) first introduced and then popularized the notion of fractals through his beautiful pictures of the Mandelbrot set (Figure 3), which is the iterative map of $Z_{n+1} = Z^2 + C$ in the complex plane, where $C = a + ib$ is a complex number and $Z_0 = 0$.

The Mandelbrot set is obtained for points that do not go to infinity (in the extended complex plane) for $n \rightarrow \infty$. These points form the large cardioid in Figure 3 and many smaller cardioids, such as the one on the right and others that are even smaller, all of which are connected with thin lines. The boundary, which is also known as a Julia set, displays extremely complicated shapes that look similar at all levels of magnification.

The Mandelbrot set is associated with the entire family of iterative maps, resulting from fixing $Z_0 = 0$ and varying C , whereas a Julia set is obtained from a single iterative map with fixed C , and an initial point near 0. If the mapping is exponential given by $Z_{n+1} = \lambda \exp(Z_n)$, where λ is complex, the resulting Julia set is not the boundary of a cardioid but a beautiful sea-horse shape.

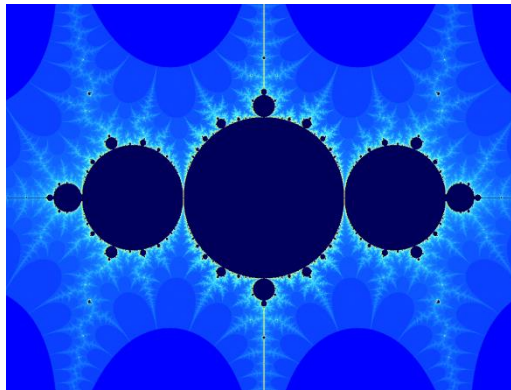


Figure 2, Julia set

Gaston Julia and Pierre Fatou, sometimes called the fathers of complex analysis, were the first to study these phenomena, and the iteration of complex exponentials has led to a new field of complex (in the sense of complex numbers) dynamics. In the domain of real numbers, perhaps the best-known fractal is the Cantor set that is obtained by removing the middle third of the real interval $[0, 1]$, then removing the middle thirds of the remaining intervals and so on. The relationship between fractals (Mandelbrot set, Julia set, Cantor set, etc.) and dynamical systems is not well having been in existence for a long time and therefore recognized and generally accepted, but a fractal is often obtained as the asymptotic remnant, or the attractor, of a chaotic dynamical system. The fractals appear in two distinct ways: a descriptive tool for studying irregular sets and forms, or a mathematical deduction, resulting from an underlying chaotic dynamic system.

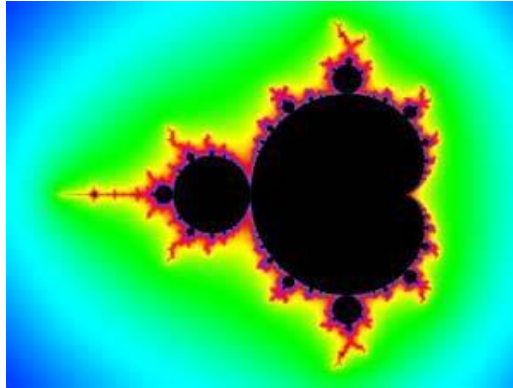


Figure 3, Mandelbrot Set

4. Fractal Dimension

To explain the concept of fractal dimension, it is necessary to understand what we mean by dimension in the first place. Obviously, a line has dimension 1, a plane dimension 2, and a cube dimension 3. But why is this? It is interesting to see students struggle to enunciate why these facts are true. One of the most known fractal is Sierpinski triangle. Let find the dimension of the Sierpinski triangle?



Fig 4 Sierpinski Triangle

They often say that a line has dimension 1 because there is only 1 way to move on a line. Similarly, the plane has dimension 2 because there are 2 directions in which to move. Of course, there really are 2 directions in a line -- backward and forward -- and infinitely many in the plane. What the students really are trying to say is there are 2 linearly independent directions in the plane. Of course, they are right. But the notion of linear independence is quite sophisticated and difficult to articulate. Students often say that the plane is two-dimensional because it has "two dimensions," meaning length and width. Similarly, a cube is three-dimensional because it has "three dimensions," length, width, and height. Again, this is a valid notion, though not expressed in particularly rigorous mathematical language.

So why is a line one-dimensional and the plane two-dimensional? Of course that both of these objects are self-similar. We may break a line segment into 6 self-similar intervals, each with the same length, and each of which can be magnified by a factor of 6 to yield the original segment. We can also break a line segment into 9 self-similar pieces, each with magnification factor 9, or 10 self-similar pieces with magnification factor 10. In general, we can break a line segment into n self-similar pieces, each with magnification factor n .

A square is different. We can decompose a square into 4 self-similar sub-squares, and the magnification factor here is 2. Alternatively, we can break the square into 9 self-similar pieces with magnification factor 3, or 16 self-similar pieces with magnification factor 4, or 25 self-similar with magnification factor 5. Clearly, the square may be broken into n^2 self-similar copies of itself, each of which must be magnified by a factor of n to yield the original figure. Finally, we can decompose a cube into n^3 self-similar pieces, each of which has magnification factor n .

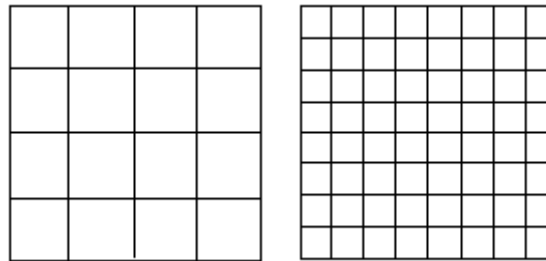


Figure 5: A square may be broken into n^2 self-similar pieces, each with magnification factor n

Now we see an alternative way to specify the dimension of a self-similar object: The dimension is simply the exponent of the number of self-similar pieces with magnification factor N into which the figure may be broken.

So what is the dimension of the Sierpinski triangle? How do we find the exponent in this case? For this, we need logarithms. Note that, for the square, we have n^2 self-similar pieces, each with magnification factor n . So we can write

$$\dim = \frac{\ln(\text{number, of, self - similar - pieces})}{\ln(\text{magnification - factor})} =$$

$$\frac{\ln n^2}{\ln n} = \frac{2 \ln n}{\ln n} = 2$$

Similarly, the dimension of a cube is

$$\dim = \frac{\ln(\text{number, of, self - similar - pieces})}{\ln(\text{magnification - factor})} =$$

$$\frac{\ln n^3}{\ln n} = \frac{3 \ln n}{\ln n} = 3$$

Thus, we take as the definition of the fractal dimension of a self-similar object

$$\dim = \frac{\ln(\text{number, of, self - similar - pieces})}{\ln(\text{magnification - factor})}$$

Now we can compute the dimension of S. For the Sierpinski triangle consists of 3 self-similar pieces, each with magnification factor 2. So the fractal dimension is

$$\dim = \frac{\ln(\text{number, of, self - similar - pieces})}{\ln(\text{magnification - factor})} =$$

$$\frac{\ln 3}{\ln 2} \approx 1,58$$

so the dimension of S is somewhere between 1 and 2.

But S also consists of 9 self-similar pieces with magnification factor 4. We have

$$\text{Fractal - Dimention} = \frac{\ln 9}{\ln 4} = \frac{\ln 3^2}{\ln 2^2} = \frac{2 \ln 3}{2 \ln 2} = \frac{\ln 3}{\ln 2} \approx 1.58$$

as before. Similarly, S breaks into 3^N self-similar pieces with magnification factors 2^N , so we again have

$$\text{Fractal - Dimention} = \frac{\ln 3^N}{\ln 2^N} = \frac{N \ln 3}{N \ln 2} = \frac{\ln 3}{\ln 2} \approx 1.58$$

Fractal dimension is a measure of how "complicated" a self-similar figure is. In a rough sense, it measures "how many points" lie in a given set. A plane is "larger" than a line, while S sits somewhere in between these two sets.

On the other hand, all three of these sets have the same number of points in the sense that each set is uncountable. Somehow, though, fractal dimension captures the notion of "how large a set is" quite nicely, as we will see below.

5. APPLICATIONS

We discuss various applications of chaos, fractals and related ideas in different disciplines that have been reported in the literature. My paper is going to give a short description on applications in the field of Mathematics, and Statistics.

5.1. Chaos and Fractals in Mathematics

The idea of defining an outer measure to extend the notion of length of an interval is of relatively recent origin in mathematics. Measures of sizes of sets originated with Borel (1895) and continued by Lebesgue (1904) as an underlying concept in the construction of an integral.

Carathodory (1914) introduced the more general concept of outer measures that is the notion of linear measure in n -dimensional Euclidean space. Hausdorff (1919) extended Carathodory's measure to non-integral dimensions. This was shown by illustrating the Hausdorff dimension of the Cantor set to be $\ln 2 / \ln 3 = 0.6309$. Sets of fractional dimension also occur in diverse branches of pure mathematics, such as the theory of numbers and nonlinear differential equations. Motivated by the theory of Brownian motion, measures of sets of curves were developed by Weiner in the 1920s that found widespread application in the theory of control and communication. An up-to-date geometric theory of sets with fractional dimension is given in Falconer (1985, 1990). Good (1941) gives an early example of an application in number theory. Investigations into asymptotic periodic behavior, transition to chaos and other dynamical consequences for ordinary differential equations are reported by different. Use of functional iterates in the theory of branching processes goes back at least to the work of Hawkins and Ulam (1944) and Good (1965). Thus, let $g(x)$ is the Laplace generating function of a sequence of probabilities $\{p_0, p_1, p_2, \dots\}$, and then the probability that an individual has k male descendants in the n th generation is given by the coefficient of x^k in $g^n(x)$ where $g^n(x) = g(g^{n-1}(x))$. Neutron multiplication in fission and fusion devices are other practical applications of functional iterations. A related area of great interest is the theory of cellular automata in which a discrete dynamical system evolves in a space of uniform grid of cells. In contrast to continuous dynamical systems modeled via differential equations or iterates of maps, theory of cellular automata specifies the system's behavior in terms of a set of local values that apply to all cells at each discrete increments of time. Study of cellular automata is a separate area in itself, with many application areas such as parallel computing, image processing and pattern recognition. Toffoli and Margolus (1987) and Preston and Duff (1984) are good introductions to this area. From the viewpoint of our discussion, theory of cellular automata is relevant for at least two reasons: first, it provides a methodology for approximating continuous systems and, second, it affords an alternative model for complex system behavior in terms of known initial conditions and simple rules of evolution. Thus, cellular automata are capable of arbitrarily complex behavior with special properties of self-replication, efficient energy transduction and so on (Wolfram, 1984, 1986; Nicolis and Prigogine, 1989). Such systems are examples of self-organization phenomena, and the field of synergetic~ is an outgrowth of study of such systems (Haken 1978). For other applications of cellular automata and a good account of the theory of functional iterations and nonlinear deterministic models, see Stein (1989). Cellular automaton models involve a great many variables, one for each cell, as opposed to models with differential equations or iterations of maps that require very few variables. On the other hand, many of the ideas and methods associated with fractals and dynamical systems, such as concepts of dimension and entropy, are applicable in the context of cellular automata. Another notion that is especially important in the latter context is algorithmic complexity. Use of notions of algorithmic complexity and their measures were first proposed by Kolmogorov (1965, 1983) and later developed by Chaitin (1987). Algorithmic complexity of a string of zeroes and ones is given by the number of bits of the shortest computer program that can generate this string. Such measures of complexity are useful for describing cellular automata and pattern formation. Rissanen (1986) has used this idea of algorithmic complexity for order determination in statistical models. The notion of complexity in a more general context was discussed in several papers by Good and summarized in Good (1977). Descriptions and characterization of complexity in spatiotemporal patterns for high-dimensional nonlinear systems is discussed by Kasper and Schuster (1987).

6. Chaos and Fractals in Statistics

It is known that deterministic model has been used for several decades for generating pseudo random numbers in stimulation experiments, the renewed interest in them is due to their possible use in modeling actual real-world process that have traditionally been studied through stochastic. There are not known methods for “fitting” a deterministic model to an actual process. A potentially important new tool resulting from the theory of chaos is the method of time –delay reconstruction of attractors from time series data. This method can give an idea about the minimum dimension of the underlying process, as well as its long term behavior. On the other hand, the reconstruction typically requires very long series, and it is sensitive to noise in the data. Noise reduction procedures specifically designed for reconstruction such as the method of Kostelich and Yorke (1987) are of great interest in this regard. These methods can complement traditional tools, such as factor and discriminate analysis, nonparametric smoothing methods and projection pursuit.

A difference route to model identification was recently suggested by Chatterjee and Yilmaz (1992). This method does not involve reconstruction, and it does not seek a deterministic model that generates an observed time series. Instead, it focuses on residuals resulting from any given stochastic time series model under consideration, such as ARIMA(p,q), and tries to determine if the residuals appear to fit a white noise process. For the latter task, the estimated fractal dimension of the residuals is compared with the fractal dimension of the white noise process.

Another potentially useful tool for the statisticians is the idea of fractal interpolation (Baransley, 1998). Given a finite number of observations, this method generates a complete path interpolating the observations and in manner consistent with self-similarity.

This idea can be useful in handling missing value in the data, and it is illustrated by Chatterjee and Yilmaz (1992). Its usefulness for prediction purpose remains to be investigated. Another recent modeling tool that seems to have been motivated by fractals and fractals dimension is the notation of fractional differencing in the ARIMA(p,d,q) models, where d is non integer. This idea can be used to model persistence (long memory) and anti-persistence (short memory) behavior. The process (0,d,0), $-1/2 < d < 1/2$ has been used by Mandelbrot and Van Ness(1968) for stimulating hydrologic data that show long-term memory.

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