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Enio Deneko
Polytechnic University of Tirana, eniodeneko@hotmail.com

Anjeza Gjini
Polytechnic University of Tirana, anjezagjini@hotmail.com

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Enio Deneko[1], Anjeza Gjini[2]
[12]Polytenchnic University of Tirana, Faculty of Civil Engineering, Tirana, Albania
eniodeneko@hotmail.com[1], anjezagjini@hotmail.com[2]

Abstract. In this paper, reference is made to the key features of ACI, EC2 and other models, regarding SLS calculations of FRP reinforcement concrete and the comparison with steel reinforcement concrete formulas, especially focusing on deflection. Mechanical characteristics of FRP materials, such as lower elastic modulus, lower ratio between Young's modulus and the tensile strength, lower bond strength of FRP bars and concrete, compared to steel reinforcement, make that SLS results determine the design of FRP reinforced concrete, based on the serviceability requirements. Different parameters influences affect the stresses in materials, maximum crack width and the allowed deflections. In this study we have calculated only the deflections of FRP-RC beams. Concrete beams reinforced with glass-fiber (GFRP) bars, exhibit large deflections compared to steel reinforced concrete beams, because of low GFRP bars elasticity modulus. For this purpose we have used equations to estimate the effective moment of inertia of FRP-reinforced concrete beams, based on the genetic algorithm, known as the Branson's equation. The proposed equations are compared with different code provisions and previous models for predicting the deflection of FRP-reinforced concrete beams. In the last two decades, a number of researchers adjusted the Branson's equation to experimental equations of FRP-RC members. The values calculated were also compared with different test results. Also it is elaborated a numerical example to check the deflection of a FRP-RC beam based on various methods of calculation of effective moment of inertia and it is made a comparison of results.

Keywords: SLS design, FRP bars, reinforced concrete beams, serviceability, deflection, effective moment of inertia, modulus of elasticity, tension stiffening.

1 Introduction

Steel reinforcing bars has not performed well in applications where members were subjects to corrosive environments. For this reasons FRP bars can be effectively used in this kind of applications because of their corrosion resistant property. The problems seem similar with those of steel RC, but solutions, limits and analytic models are different, because of the very large band of FRP bars on the market, with a large variety of mechanical characteristics. There are many types of fibers including glass (GFRP), carbon (CFRP) and aramid (AFRP), with different grades of tensile strength and modulus of elasticity. The behavior of FRP-RC beams differs from steel reinforcing beams, because FRP bars display a linear elastic behavior up to the point of failure and do not demonstrate ductility. Also the bond strength of FRP bars and concrete is lower than that of steel bars, leading to an increase in the depth of cracking, a decrease of stiffening effect, and so an increase of the deflection of FRP-RC beams for an equivalent cross-section of reinforcement of steel reinforced concrete beams.
FRP-RC beams have lower elastic modulus than steel bars, for example, the modulus of elasticity of GFRP bars is only 20-25% of that in steel bars. Because of this low modulus of elasticity, the deflection criterion may control the design of long FRP-RC beams. Consequently a method is needed in order to know the expected service load deflections with a high degree of accuracy.
Only some countries have developed a code in FRP-RC design and these codes are still in preparation phase, so is very difficult to operate in this new field. Following a presentation of the key concepts, the paper discusses topics for future implementation and sample applications.

2 Methodology

FRP are anisotropic materials and are characterized by high tensile strength with no yielding in the direction of the reinforcing fibers. An FRP-RC member is designed on its required strength and then checked for serviceability and ultimate state criteria (e.g., deflection, crack width, fatigue and creep rapture). In most instances, serviceability criteria will control the design.
Safety checks for FRP-RC at SLS are more important than that for steel reinforced concrete because of the mechanical characteristics of FRP and especially the low ratio between Young's modulus and the tensile strength of FRP reinforcement.
In this paper are taken into account different models for the deflection design of FRP-RC beams, some of them already used for steel-RC, and some others are brand new ones, used only for FRP-RC beams. FRP are anisotropic materials and are characterized by high tensile strength with no yielding in the direction of the reinforcing fibers.

3 Deflection and deformation

A well-known deflection model for Steel-RC is proposed by ACI and EC2, later modified for FRP-RC. In both of them the M_{cr}/M_{max} ratio is very important for the deflection design, where: M_{max} is the maximal moment acting on the examined element; M_{cr} is the cracking moment at the same cross section of M_{max}. This deflection model simulates the real behavior of the structure by taking into account cracking, but not the tension-stiffening effect of concrete.
ACI 318-95 is based on Branson [1968-1977] formulae of the effective moment of inertia for Steel-RC:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 \cdot I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] \cdot I_{cr} \leq I_g \,. \qquad \textbf{(1)}$$

where, I_e is the effective moment of inertia, M_{cr} is the cracked section moment, I_g is the total moment of inertia, M_a is the maximum moment in member at the deflection stage, I_{cr} is the cracked section moment of inertia.

ACI 440R-96 proposed new formulae of the effective moment of inertia for FRP-RC:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 \cdot \beta_d \cdot I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] \cdot I_{cr} \leq I_g \,. \qquad \textbf{(2)}$$

where, β_d is a reduction coefficient related to the reduced tension-stiffening exhibited by FRP-RC members. Based on evaluation of experimental results

$$\beta_d = \frac{1}{5}\left(\frac{\rho_f}{\rho_{fb}}\right) \leq 1 \,. \qquad \textbf{(3)}$$

This equation is valid only if $M_a \geq M_{cr}$. If $M_a \leq M_{cr}$, than $I_e = I_g$ and if $M_a \approx M_{cr}$, or slightly less, than $I_e = I_{cr}$, because shrinkage and temperature may cause section cracking.
Some other models based on ACI give this formula:

$$I_e = \frac{I_g}{\beta} \cdot \left(\frac{M_{cr}}{M_{max}}\right)^3 + \alpha \cdot I_{cr} \left[1 - \left(\frac{M_{cr}}{M_{max}}\right)^3\right] \leq I_g \,. \qquad \textbf{(4)}$$

where, α and β are coefficients of bond properties of FRP and

$$\frac{1}{\beta} = \alpha^* \left(\frac{E_{frp}}{E_s} + 1\right) \leq 1 \,. \qquad \textbf{(5)}$$

where, α^* is a coefficient that increases with the bond quality. When no experimental results are given than α^*= 0, 5 and α = 1.

EuroCode 2 propose a simplified model for Steel-RC

$$v = v_1 \cdot \gamma + v_2 \cdot (1-\gamma) \text{ where } \gamma = \beta \cdot \left(\frac{M_{cr}}{M_{max}}\right)^m . \tag{6}$$

In this equation v_1 and v_2 are calculated taking into account that the moment of inertia of the section is constant and respectively I_1 and I_2 (or I_g and I_{cr}). For steel bars the coefficients β and m, including the tension-stiffening effect of concrete are to be taken as given in the table.

Table 1. Coefficient β and m for steel bars

Model	β	m
EC2	1,0	2,0
CEB	0,8	1,0

If other materials are used (e.g. FRP bars), these coefficients must be reconsidered through experimental tests.
EC2 hasn't proposed yet a model for FRP-RC deflection design, but the Italian Code CNR-DT 203/2006, based on experimental tests, shows that the model for EC2 can be deemed suitable for FRP-RC too. Therefore the EC2 equations to compute deflection "f" must be reconsidered:

$$f = f_1 \cdot \beta_1 \cdot \beta_2 \cdot \left(\frac{M_{cr}}{M_{max}}\right)^m + f_2 \cdot \left[1 - \beta_1 \cdot \beta_2 \cdot \left(\frac{M_{cr}}{M_{max}}\right)^m\right]. \tag{7}$$

where, f_1 gives the deflection of non-cracked section, f_2 gives the deflection of cracked section, β_1 = 0,5 is the coefficient of bond properties of FRP bars, β_2 is the coefficient of the duration of loading and is taken $\beta_2=1$ for short-time loads and $\beta_2=0{,}5$ for long-time cycling loads, M_{max} is the maximum moment acting on the examined element, M_{cr} is the cracking moment calculated at the same cross section of M_{max}, m = 2.

In the last two decades a number of researchers tried to adjust the Branson's equation, comparing to experimental results of FRP-RC member tests. The experimental results show that Branson's equation overestimated the moment of inertia I_e and underestimates the deflection, because the Branson's equation was calibrated for RC beams where $I_g/I_{cr} \le 3$ [Bischoff 2005], but not for most members that have $5 \le \frac{I_g}{I_{cr}} \le 25$ [Bischoff 2009]. Also the bond behavior between FRP bars and concrete differs from the bond behavior between steel and concrete, so the tension stiffening effect must be re-evaluated.

Benmokrane [1996], added two reduction factors and adjusted this equation:

$$I_e = \alpha \cdot I_{cr} + \left(\frac{I_g}{\beta} - \alpha \cdot I_{cr}\right) \cdot \left(\frac{M_{cr}}{M_{max}}\right)^3 \le I_g . \tag{8}$$

From experimental data α=0.84 and β=7, because of the nature of FRP reinforcement, with larger deflection and greater reduction of compressed concrete section when applied M_{cr}.

Faza and Gangarao [1992], proposed a model for two concentrated point loads based on the assumption that a concrete section between the point loads is fully cracked, while the end sections are partially cracked.

$$I_m = \frac{23 \cdot I_{cr} \cdot I_e}{8 \cdot I_{cr} + 15 \cdot I_e} \text{ Where } I_e = \left(\frac{M_{cr}}{M_a}\right)^3 \cdot I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] \cdot I_{cr} . \quad \textbf{(9)}$$

The maximum deflection is calculated as follows:

$$\Delta_{max} = \frac{23 \cdot P \cdot L^3}{648 \cdot E_c \cdot I_e} . \quad \textbf{(10)}$$

Toutanji and Saafi [2000] adjusted the ratio M_{cr}/M_a to take into account the modulus of elasticity of FRP bars *(E_{frp})* and the reinforcement ratio *(ρ_f)*. They took a set of 13 GFRP-RC beams with a ratio $13 \leq \frac{I_g}{I_{cr}} \leq 25$. The model proposed was:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^m \cdot I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^m\right] \cdot I_{cr} \leq I_g . \quad \textbf{(11)}$$

where: $m = 6 - \frac{10 \cdot E_f}{E_s} \cdot \rho_f$, if $\frac{E_f}{E_s} \cdot \rho_f < 0.3$ and $m = 3$, if $\frac{E_f}{E_s} \cdot \rho_f \geq 0.3$

Brown and Bartholomew [1996], used quite the same model based on tests of two GFRP-RC beams with the ratio $\frac{I_g}{I_{cr}} \approx 11$ and used $m = 5$, while *A-Sayed [2000]* proposed $m = 5{,}5$.

ACI 440.1 R-01 adopted the modification proposed by *GAO [1998]:*

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 \cdot \beta_d \cdot I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] \cdot I_{cr} \leq I_g . \quad \textbf{(12)}$$

where, β_d = 0,6 based on Masmoudi (1998) and Theriault & Benmokrane [1998] studies. They recommended,

$$\beta_d = \alpha_b \left(\frac{E_{frp}}{E_s} + 1\right) \leq I_g . \quad \textbf{(13)}$$

where, α_b is a bond dependented coefficient: α_d =0.5 for GFRP [GAO 1998] and later based on experimental tests of 48 GFRP-RC and the amount of the longitudinal reinforcement:

$$\alpha_b = 0{,}064 \cdot \left(\frac{\rho_f}{\rho_{fb}}\right) + 0{,}13 . \quad \textbf{(14)}$$

Recently *Rafi and Nadjai [2009]*, introduced γ factor, that reduces the portion of cracked moment of inertia:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 \cdot \beta_d \cdot I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] \cdot \frac{I_{cr}}{\gamma}{}_{cr} \leq I_g . \quad \textbf{(15)}$$

$$\gamma = 0{,}86 \cdot \left(1 + \frac{E_f}{400}\right) . \quad \textbf{(16)}$$

$$\beta_d = \frac{1}{5}\left(\frac{\rho_f}{\rho_{fb}}\right) . \quad \textbf{(17)}$$

ISIS Canada [2001], based on *Ghali and Azarnejad [1999]*, when service load level is less that cracked moment, M_{cr}, the immediate deflection can be evaluated using the transformed moment of inertia, I_t, instead of effective moment of inertia, I_e , used when service moments exceed the cracked moment.

Mota [2006] examined a number of the suggested formulations for I_e and found an equation that provided the most conservative results over the entire range of experimental results of test specimens.

$$I_e = \frac{I_t \cdot I_{cr}}{I_{cr} + \left[1 - \frac{1}{2}\left(\frac{M_{cr}}{M_a}\right)^2 \right] \cdot (I_t - I_{cr})} \ . \quad \textbf{(18)}$$

Where, I_t is the moment of inertia of a non-cracked concret section, and

$$I_{cr} = \frac{b \cdot (k \cdot d)^3}{3} + n_{frp} \cdot A_{frp} \cdot (d - k \cdot d)^2 \ . \quad \textbf{(19)}$$

$$k = \sqrt{\left(\rho \cdot n_{frp}\right)^2 + 2 \cdot \rho \cdot n_{frp}} - \rho \cdot n_{frp} \ . \quad \textbf{(20)}$$

$$n_{frp} = \frac{E_{frp}}{E_c} \ . \quad \textbf{(21)}$$

$$\rho = \frac{A_{frp}}{b \cdot d} \ . \quad \textbf{(22)}$$

where, b represent the width of cross-section (mm) and d the depth to FRP layer (mm).

CAN/CSA-S806 [2002] used *Razqapur* methodology which assumes that tension stiffening is insignificant in cracked regions on FRP-RC beams, using $E_c \cdot I_g$ when $M_a < M_{cr}$, and $E_c \cdot I_{cr}$ when $M_a > M_{cr}$, to integrate the curvature *M/EI* along the beam span. This leads to a simple expression for beam deflection δ_{max}, for a four-point bending configuration with two point loads at a distance *a* from the supports, assuming L_g, the distance that the beam is uncracked:

$$\delta_{max} = \frac{PL^3}{24 E_c I_{cr}} \left[3\left(\frac{a}{L}\right) - 4\left(\frac{a}{L}\right)^3 - 8\left(1 - \frac{I_{cr}}{I_g}\right)\left(\frac{L_g}{L}\right)^3 \right]. \quad \textbf{(23)}$$

Saikia [2007], used the same expression in his tests and found the same agreement with his experimental data.
Bischoff [2005], *Bischoff [2007a]*, *Bischoff and Scanlon [2007]*, proposed an equation derived from integration of curvatures along the beam taking into account the tension-stiffening effect:

$$I_e = \frac{I_{cr}}{1 - \left(1 - \frac{I_{cr}}{I_g}\right) \cdot \left(\frac{M_{cr}}{M_a}\right)^2} \ . \quad \textbf{(24)}$$

Abdalla, *El-Badry* and *Rizkalla* introduced a model similar to EC2-CEB, suggesting α=0.85 and β=0.5

$$v = \left(\frac{M_{cr}}{M_a}\right) \cdot \beta \cdot v_1 + \left[1 - \beta\left(\frac{M_{cr}}{M_a}\right) \right] \cdot \alpha \cdot v_2 \ . \quad \textbf{(25)}$$

But *Abdalla [2002]* gave also a model based on ACI:

$$I_e = \frac{I_g \cdot I_{cr}}{I_{cr} \cdot \xi + 1{,}15 \cdot I_g (1 - \xi)} \ . \quad \textbf{(26)}$$

where, $\xi = \frac{0{,}5 M_{cr}}{M_a}$. This equation has a coefficient of 1.15 (or better 1/0.85), that takes into account the reduction of tension-stiffening effect in the fully cracked FRP concrete section.
The Norwegian Code (Eurocrete), calculates the deflections taking: $I_m = I_{cr}$ (so the the section considered fully cracked).

The Japanese Code (JSCE), consent only the use of CFRP bars with Young modulus comparable with the Young modulus of the steel, and suggests conserving the same models used for traditional steel reinforced beams.

4 The Result Comparison

To compare the result, we have made calculation for a simply supported, normal weight interior beam with a span length $l = 4\ m$ and $f_c' = 30\ MPa$. It is designed to carry a service live load of $w_{LL} = 6\ kN/m$ and a superimposed service dead load of $w_{SDL} = 3\ kN/m$. The cross section of the beam is to be taken as 250 mm x 400 mm. 4 Ø 16 GFRP bars are selected as main beam reinforcement and Ø 9.5 GFRP bars are selected as shear beam reinforcement. Material properties of GFRP bars are: tensile strength f_{fu}^*= 320 MPa, rupture strain ε_{fu}^*=0.014 and Modulus of elasticity E_f = 44 800 MPa. The results are given in the table below.

Table 2. Results taken from different methods

Reference	I_e	Δ_{LT}
ACI 318R-95 (1995), Branson	$13{,}08 \cdot 10^8\ mm^4$	$1{,}36 mm$
Benmokrane (1996)	$1{,}887 \cdot 10^8\ mm^4$	$9{,}45 mm$
ACI 440.1R-03 (2003)	$8{,}011 \cdot 10^8\ mm^4$	$2{,}22 mm$
Yost (2003) based on ACI	$3{,}128 \cdot 10^8\ mm^4$	$5{,}70 mm$
ACI 440.1R -06 (2006)	$2{,}641 \cdot 10^8\ mm^4$	$6{,}75 mm$
Rafi & Nadjai (2009)	$2{,}642 \cdot 10^8\ mm^4$	$6{,}75 mm$
EC2-CEB, Italian Code CNR-DT 203/2006		$8{,}72 mm$
Toutanji & Saafi (2000)	$12{,}89 \cdot 10^8\ mm^4$	$1{,}39 mm$
Alsayed Model A (2000)	$12{,}83 \cdot 10^8\ mm^4$	$1{,}38 mm$
Alsayed Model B (2000)	$1{,}649 \cdot 10^8\ mm^4$	$10{,}82 mm$
Bischoff (2005,2007) & Scanlon (2007)	$11{,}81 \cdot 10^8\ mm^4$	$1{,}51 mm$
Abdalla based on ACI (2002)	$2{,}075 \cdot 10^8\ mm^4$	$8{,}60 mm$
Abdalla, Rizkalla & El Badry (EC2)		$5{,}34 mm$
ISIS Canada (2001) & Mota (2006)	$2{,}514 \cdot 10^8\ mm^4$	$7{,}48 mm$
Hall & Ghali (2000)	$2{,}514 \cdot 10^8\ mm^4$	$7{,}48 mm$

According to Branson's equation, the effective moment of inertia I_e , at different levels of loading, takes values between the moment of inertia of non-cracked gross concrete section I_g , and the moment of inertia of the cracked section I_{cr} . Here, the effective moment of inertia I_e is always less than I_{cr} , but it approaches it after cracking occurs.

For this reason, is calculated the mid span displacements of a GFRP reinforced concrete beam, based on different equations for effective moments of inertia I_e , using different values of span lengths, different levels of loading (only different service dead loads, while the service live load is maintained constant), in normal reinforced ratio ($\frac{\rho_{frp}}{\rho_{fb}} \approx 1$) and in high reinforced ratio ($\frac{\rho_{frp}}{\rho_{fb}} \geq 2.5$). All the results are included in comparative charts in order to find out some theoretical conclusions.

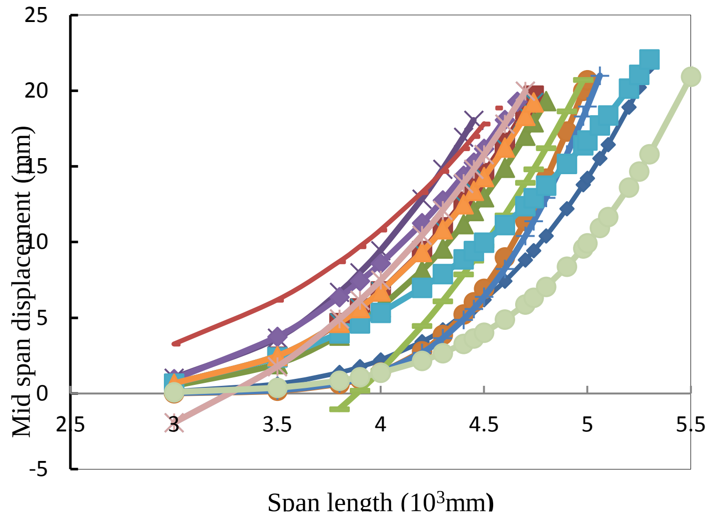
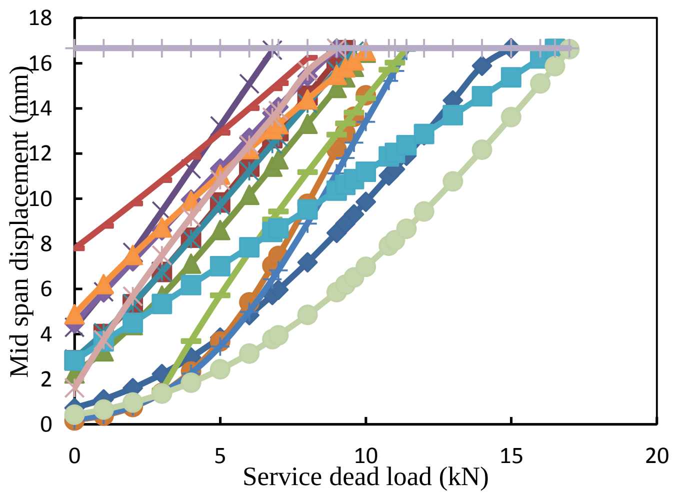


Fig. 1. Relation between span length and mid span displacement



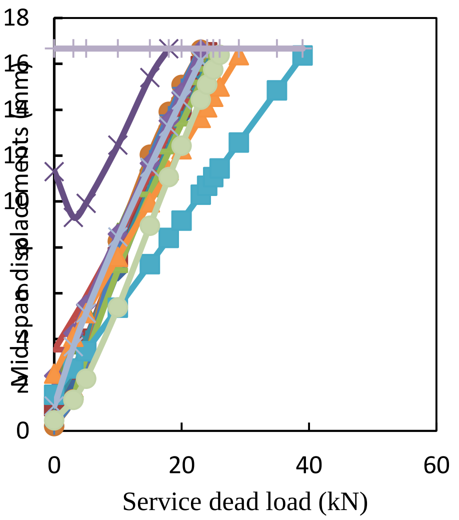


Fig. 2. Relation between service dead load and mid span displacement for $\frac{\rho_{frp}}{\rho_{fb}} \approx \mathbf{1}$

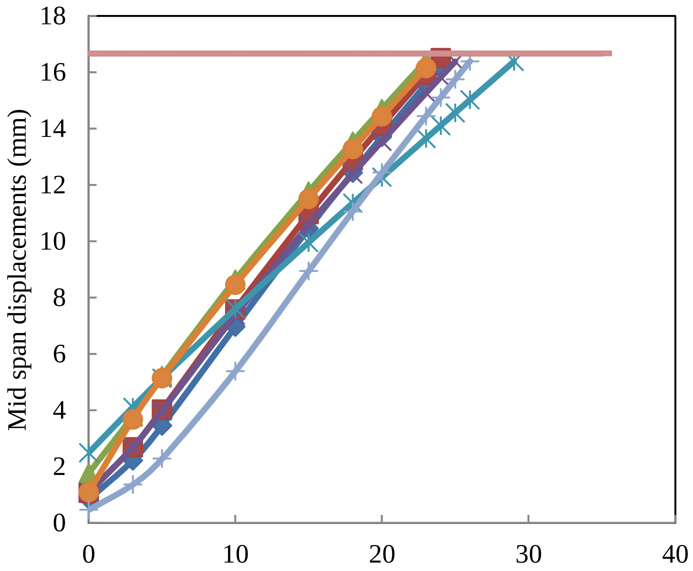


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Fig. 3. Relation between service dead load and mid span displacement for $\frac{\rho_{frp}}{\rho_{fb}} \approx \mathbf{2.53}$

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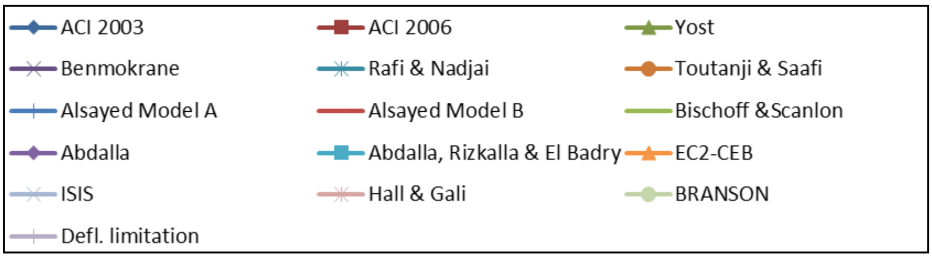

Fig. 5. Legend of methods

5 Summary and results

As shown in figure 1, all methods lead to quite the same result: increasing the span length, we get bigger mid span displacements until the maximum allowed deflection is reached. But we can see that the Branson's equation is just an envelope for the other methods, for short beams we get smaller deflections than other methods and for longer ones we get bigger displacement than other methods. This because Branson's equation doesn't takes into account the type of FRP used as reinforcement, so it hasn't used any reduction factor based on the $\frac{E_{frp}}{E_s}$ ratios, or even on the quantity of the reinforcement used so the $\frac{\rho_{frp}}{\rho_{fb}}$ ratios, as other methods do. So this equation is more generalized and conservative.

A tentative was made using ACI 440.1R -03 [2003], where a reduction factor based on the $\frac{E_{frp}}{E_s}$ ratios is taken into account, but not the reinforcement ratio, or ACI 440.1R -06 [2006] where only the reinforcement ratio is taken into account. The results show that the second part of Branson's equation outweighs the first part in concrete beams reinforced with FRP bars, so the most important of the two reduction coefficients is the one based on reinforcement ratio. Yost (2003) based on ACI and Rafi & Nadjai [2009] are the most reliable methods because they use both reduction factors, but also ACI 440.1R -06 [2006] gives satisfactory results. For this reason, the three methods, based on ACI code, gives more accurate results for beams with lower reinforcement ratios.

Also, Hall& Gali, ISIS Canada [2001] and the Canadian Code [CAN/CSA-S806 2002], using different equation instead of Branson's one, where is introduced I_t, the moment of inertia of a non-cracked section transformed to concrete taking into account not only the reinforcement and modulus of elasticity ratios, but also a coefficient characterizing the bond properties of reinforcement bars, gives accurate and similar results as the others above. The only discrepancy of these methods (including Bischoff &Scanlon 2007), is the negative values of mid span displacements for very short beams because those take into account also the tension-stiffening effect.

EC2-CEB, Italian Code CNR-DT 203/2006 gives good results that don't depend on the effective moment of inertia, but only on displacements of non-cracked cross concrete section and the cracked section. It takes into account the bond properties of reinforcement and the type of load: short time loads or long-time cycling loads, but doesn't depend on reinforcement and modulus of elasticity ratios.

Figure 2 shows the relation between service dead load and mid span displacements for $\frac{\rho_{frp}}{\rho_{fb}} \approx 1$. For normal reinforcement ratio $\frac{\rho_{frp}}{\rho_{fb}} \leq 2.5$, at different levels of loading, I_e is still less than I_{cr} , regardless the chosen method we use to calculate the I_e . As shown in figure 3, if reinforcement ratios increases, especially for $\frac{\rho_{frp}}{\rho_{fb}} > 2.5$, than deflections decreases. This happens because the cracking moment of the beam increases and few cracks appear at the same level of loading. The compressive strength of concrete increases too, but this effect on beam's deflection is not considerable in high reinforcement ratios. The increase of $I_{cr,}$ for high levels of loading and reinforcement ratios may exceed the values of I_e .

It is interesting the fact that, the deflections calculated using most of these methods, are more consistent with each other in high levels of loading and reinforcement ratios. This occurs in most of methods where the minimum effective moments of inertia I_e are quite equal to the cracked moment of inertia I_{cr} . The deflections seem to be the same for quite all methods. Some deviations are presented by Benmokrane where $I_e < I_{cr}$ independently from the level of loading and Abdalla,Rizkalla & El Badry (based on EC2) and also EC2, because they don't use I_e .

In figure 4 are selected the most used and reliable methods for better comparison.

6 Conclusions

Results show that deflection values calculated using ACI 440.1R -06 [2006] code are more accurate than those using ACI 440.1R -03 [2003] for beams with low reinforcement ratio but not much satisfactory if $\frac{\rho_{frp}}{\rho_{fb}} > 2.5$, especially if $\frac{\rho_{frp}}{\rho_{fb}} > 3$.

Based on a lot of experimental results, the reinforcement ratio and the elastic modulus of FRP bars are the most significant variables to calculate deflections. Yost [2003] based on ACI and Rafi & Nadjai [2009] and also ACI 440.1R -06 [2006] are the most reliable methods because they use both reduction factors taking into account these variables and give good results, so this three methods, based on ACI code, gives more accurate results for beams with low and high reinforcement ratios.

Also, Hall& Gali, ISIS Canada [2001] and the Canadian Code (CAN/CSA-S806 2002), using a different method instead of Branson's equation and taking into account also the bond properties of reinforcement bars gives good results and after all the deflections calculated using these methods based on Canadian Code are more conservative of those based on ACI code.

All the equations can better predict deflections for $I_e < I_{cr}$, especially at high levels of loading and reinforcement ratios. For $I_e \geq I_{cr}$, we don't get reliable results because we get very high deflections for low levels of loading.

In this study is used only a type of GFRP bar but exist different types of FRP reinforcements with very large properties, so different results are taken based on different methods. For this reason a lot of researchers, based on a large numbers of experimental tests are modifying and optimizing Branson's equation, so that the predicted values of deflection approach the experimental values. New models are going to be developed based on experimental results and elaborated genetic algorithm used to evaluate the effects of several parameters and reevaluate the power m in Branson's equation as a lot of researchers are doing like Toutanji & Saafi [2000], Alsayed Model A [2000], Mousavi& Esfahani [2011], etc.

References

1. Hota V. S. GangaRao, Narendra Taly, P. V. Vijay: Reinforced Concrete Design With FRP Composites, CRC Press Taylor & Francis Group, Boca Raton, Florida (2007)
2. American Concrete Institute (ACI): Guide for the Design and Construction of Concrete
3. American Concrete Institute, ACI Committee 440, ACI 440.1R-01, Farmington Hills,MI: Reinforced With FRP Bars (2001)
4. American Concrete Institute (ACI): Guide for the Design and Construction of Concrete
5. Reinforced With FRP Bars, ACI Committee 440, ACI 440.1R-03, Farmington Hills, MI: American Concrete Institute (2003)
6. Branson, D.E.: Deformation of Concrete Structures, New York: McGraw-Hill (1977)
7. ISIS Canada: Intelligent Sensing for Innovative Structures, Reinforcing Concrete Structures With Fiber-Reinforced Polymers, Design Manual No. 3, Version 2 (2007)
8. Lawrence C. Bank: Composites for construction, Structural Design with FRP Materials, John Wiley & Sons, Inc., Hoboken, New Jersey (2006)
9. CEB-FIB Bulletin: FRP reinforcement for RC structures (2006)
10. Nanni A. and Faza S.: Design and construction of concrete reinforcement with FRP bars , an emerging technology (2001)
11. Consiglio Nazionale delle Ricercce: Istruzioni per la Progettazione, l'Esecuzione ed il Controllo di Strutture di Calcestruzzo armato con Barre di Materiale Composito Fibrorinforzato, CNR-DT 203/ (2006)
12. Benmokrane, B., Challal, O., and Masmoudi, R.: Flexural response of concrete beams reinforced with FRP bars, ACI Struct. J., 93, 1, 46–55, January-February (1996)
13. Abdalla, H. A. : Evaluation of deflection in concrete member reinforced with fiber reinforced polymer , FRP bars (2002)
14. Yost,J.R.,Gross,S.P.,and Dinehart,D.W.: Effective moment of inertia for glass fiber-

reinforced polymer-reinforced concrete beams" ACI Struct. J. , 100(6), 732-739. (2003)
15. Canadian Standard Association (CSA).: Design and construction of building components with fiber-reinforced polymers, Concrete Design Handbook : Canadian Standard S806-02, CSA, Toronto, (2002)
16. Bischoff, P.H.: Deflection calculation of FRP reinforced concrete beams on modifications to the existing Branson's equation. (2007)