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COMPUTER AIDED ANALYSIS OF KINEMATICS AND KINETOSTATICS OF SIX-BAR LINKAGE MECHANISM THROUGH THE CONTOUR METHOD

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Abstract

In this paper is presented a six-bar linkage mechanism of the pump for oil extrusion. In this mechanism are introduced even higher kinematic pairs. Dimensions and other incoming links will be adopted as necessary. For the six-bar linkage mechanism will be carried out the kinematic analysis and for all linkages will be presented the displacement, velocity and acceleration. The analysis will be performed by MathCad software, while kinetostatic analysis will be carried out using Contour Method, comparing results of two different software's MathCAD and Working Model. The simulation parameters will be computed for all points of the contours of mechanism.

Keywords: mechanism, contour, displacement, speed, acceleration

This paper has been realised with two application software's: MathCad and Working Model. In this paper is presented a four-bar linkage ABCD and a simple crank mechanism DEF as shown in the figure below. In this mechanism firstly is determined DF_y and θ_5 . Derivative DF_y represents the speed of the slider F. From the given picture $\theta_2 = 0,1,\dots,360^\circ$ should be determined. The masses are adopted, since the moments of inertia need to calculate. The kinematic part of the paper will be completed by finding the velocities and accelerations of each point A, B, C, D, E, F for the centres from C_1 to C_6 . In this way are determined the angular accelerations and velocities of the linkages 2, 3, 4 and 5. Whereas for the kinetostatic part will be determined the reaction forces of the points: $X_A, Y_A, X_B, Y_B, X_C, Y_C, X_D, Y_D, X_E, Y_E, X_F, Y_F, N = X_{F6}$ and M_{tr} which are acting on the leading link 2. In the picture are shown 5 bodies: AB, BC, right triangle, rod EF, slider bar F.

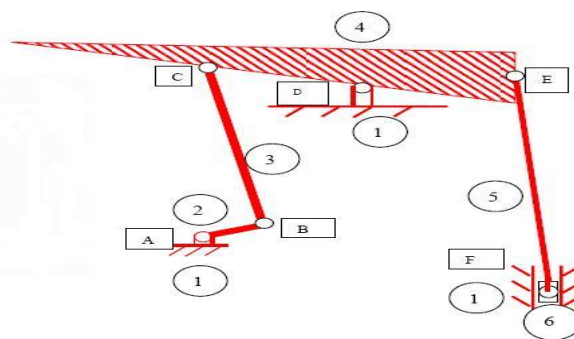


Figure 1: Six bar-linkage mechanism ABCDEF

Given data:

$$\theta_2 := 0,5 \text{deg}.. 360 \text{deg}$$

$$\omega_2 := 2$$

$$r_2 := 1.7$$

$$r_3 := 6.9$$

$$r_{4a} := 4.2$$

$$r_{4b} := 4.0$$

$$r_5 := 9.2$$

$$r_D := \sqrt{4.2^2 + 6.4^2}$$

$$\alpha := \arccos\left(\frac{4.2}{\sqrt{4.2^2 + 6.4^2}}\right) \quad \frac{\alpha}{\text{deg}} = 56.725$$

Determination of angle α of r_D with x_1 :

Linkage I

$$\vec{r}_2 + \vec{r}_3 + \vec{r}_{4a} = \vec{r}_D$$

From then designed outline at x and y we have the following initial conditions:

Given

$$r_2 \cdot \cos(\theta_2) + r_3 \cdot \cos(\theta_3) + r_{4a} \cdot \cos(\theta_4) = r_D \cdot \cos(\alpha)$$

$$r_2 \cdot \sin(\theta_2) + r_3 \cdot \sin(\theta_3) + r_{4a} \cdot \sin(\theta_4) = r_D \cdot \sin(\alpha)$$

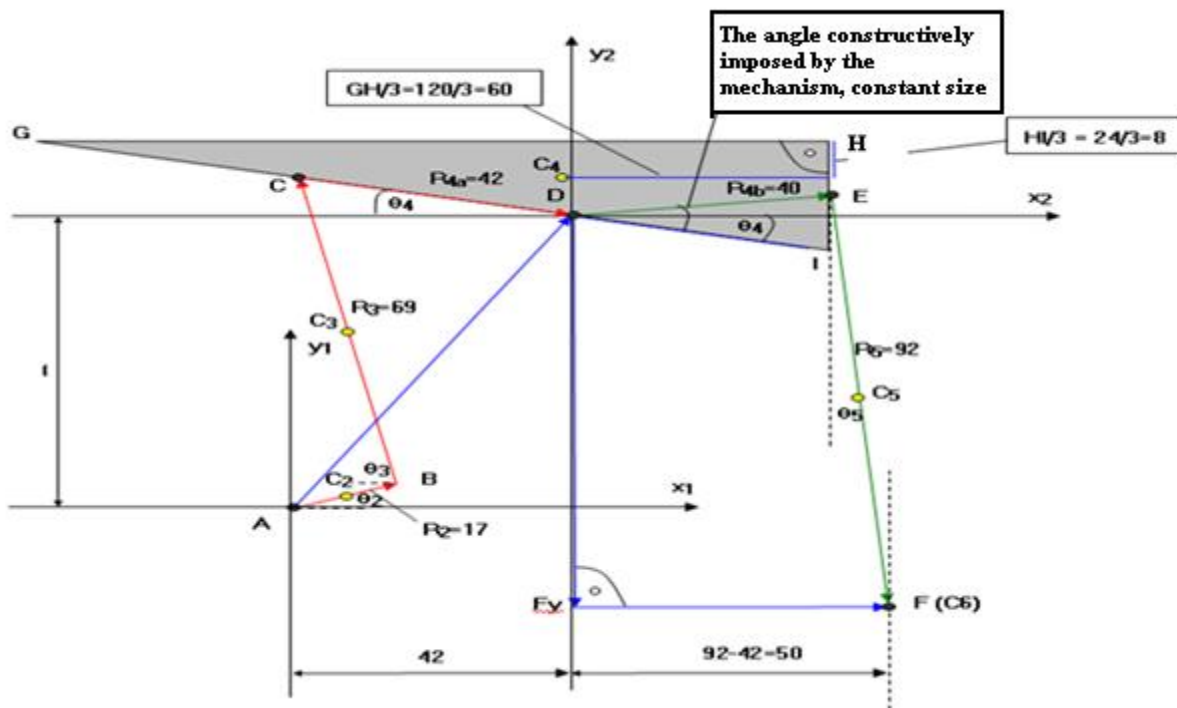


Figure 2: Four-bar linkage $ABCD$ and crank mechanism DEF

As shown in the Figure 2, there are 14 unknown sizes, 12 equations for the first four bodies, $dy(x, y)$ for the slider 14 equations.

1.1 The analysis of the positions, velocities and accelerations of six-bar linkage mechanism

In the figure 3 is presented the velocity plan for the points A, B, C and D . Also are presented angular velocities and angular accelerations for the points A, B and C . After finding the velocities and accelerations of these points, the displacements, velocities and accelerations of these points with angles θ_3 and θ_4 in function of (θ_2) are shown graphically. Outline is shown in this form:

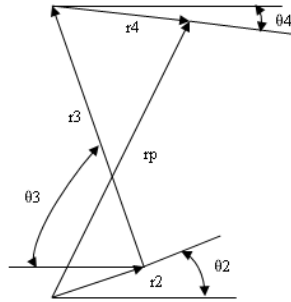


Figure 3: Velocity plan for the points A, B and C

Given $\theta_3 := 180 \cdot \text{deg}$ $\theta_4 := 90 \cdot \text{deg}$

Ekuacioni vektorial për konturin I:

$$\begin{pmatrix} r_2 \cdot \cos(\theta_2) \\ r_2 \cdot \sin(\theta_2) \\ 0 \end{pmatrix} + \begin{pmatrix} r_3 \cdot \cos(\theta_3) \\ r_3 \cdot \sin(\theta_3) \\ 0 \end{pmatrix} + \begin{pmatrix} r_4 \cdot \cos(\theta_4) \\ r_4 \cdot \sin(\theta_4) \\ 0 \end{pmatrix} = \begin{pmatrix} r_p \cdot \cos(\alpha) \\ r_p \cdot \sin(\alpha) \\ 0 \end{pmatrix} \dots\dots\dots(1)$$

Zgj(θ_2) := Find(θ_3, θ_4)

$\theta_3(\theta_2) := \text{Zgj}(\theta_2)_0$

$\theta_4(\theta_2) := \text{Zgj}(\theta_2)_1$

$$\beta := \text{asin}\left(\frac{2.4}{\sqrt{1.2^2 + 2.4^2}}\right) \quad \frac{\beta}{\text{deg}} - \frac{-\theta_4(0)}{\text{deg}} = 59.478$$

$$\frac{\theta_3(0)}{\text{deg}} = 104.177$$

$$\frac{\theta_4(0)}{\text{deg}} = -3.957$$

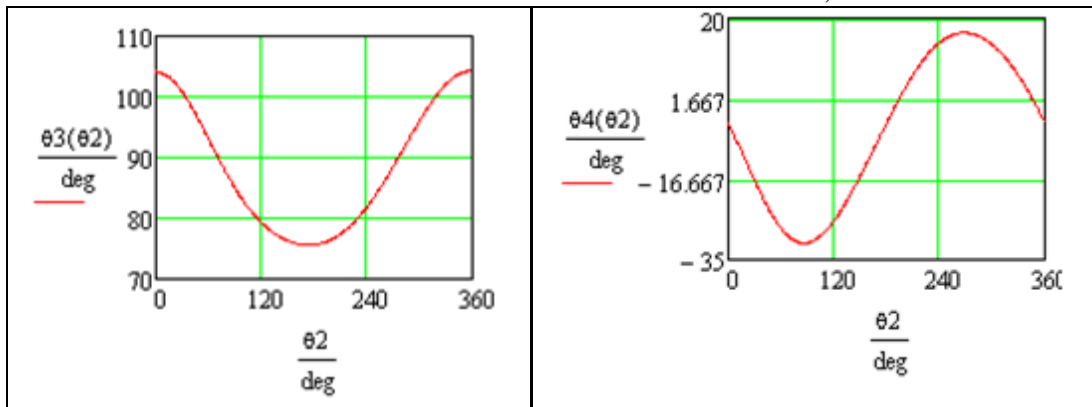
$$\omega_3(\theta_2) := \frac{d}{d\theta_2} \theta_3(\theta_2) \cdot \omega_2$$

$$\omega_4(\theta_2) := \frac{d}{d\theta_2} \theta_4(\theta_2) \cdot \omega_2$$

$$\varepsilon_3(\theta_2) := \frac{d^2}{d\theta_2^2} \theta_3(\theta_2) \cdot \omega_2^2$$

$$\varepsilon_4(\theta_2) := \frac{d^2}{d\theta_2^2} \theta_4(\theta_2) \cdot \omega_2^2$$

After calculation of the values by MathCad software, in the following are shown graphically the values of positions, velocities and accelerations respectively for $\theta_3, \omega_3, \varepsilon_3$ and $\theta_4, \omega_4, \varepsilon_4$:



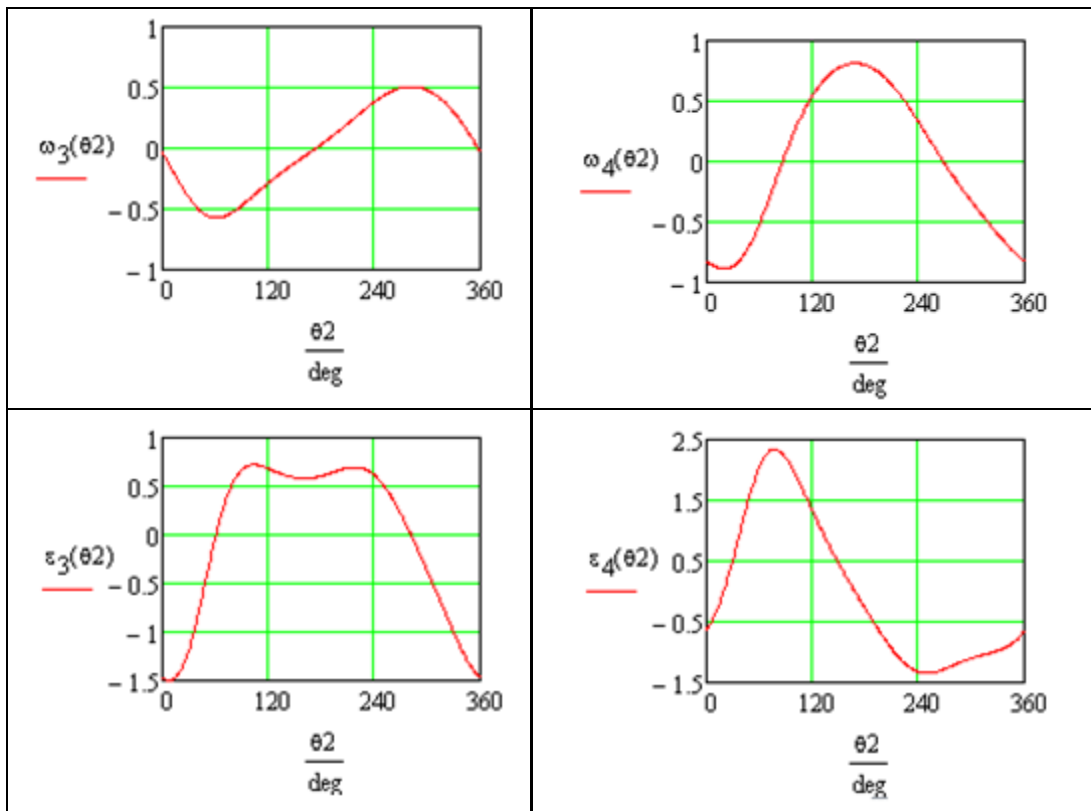


Figure 4: Diagrams for positions, velocities and accelerations

1.2 Position of the point B

The position of the point B is calculated in direction of x and y :

$$x_B(\theta_2) := r_2 \cdot \cos(\theta_2)$$

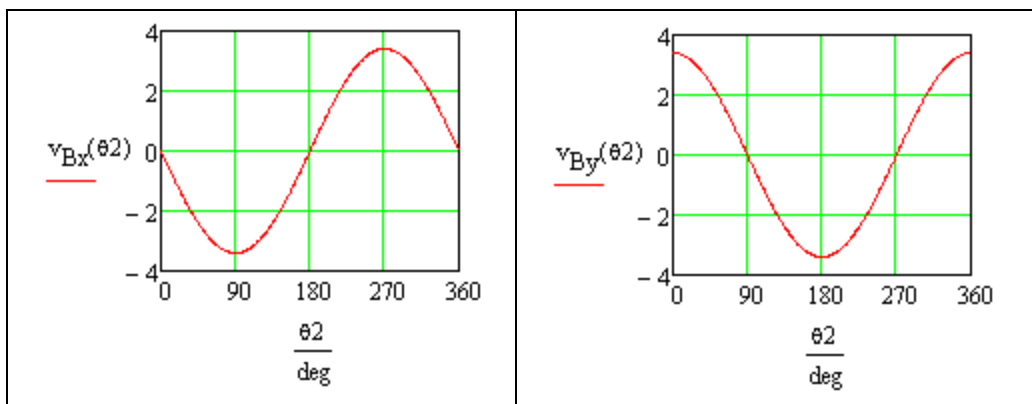
$$y_B(\theta_2) := r_2 \cdot \sin(\theta_2)$$

$$v_{Bx}(\theta_2) := -r_2 \cdot \sin(\theta_2) \cdot \omega_2$$

$$v_{By}(\theta_2) := r_2 \cdot \cos(\theta_2) \cdot \omega_2$$

$$v_B(\theta_2) := \sqrt{v_{Bx}(\theta_2)^2 + v_{By}(\theta_2)^2}$$

$$\sqrt{9.2^2 + 4.0^2} = 10.032$$



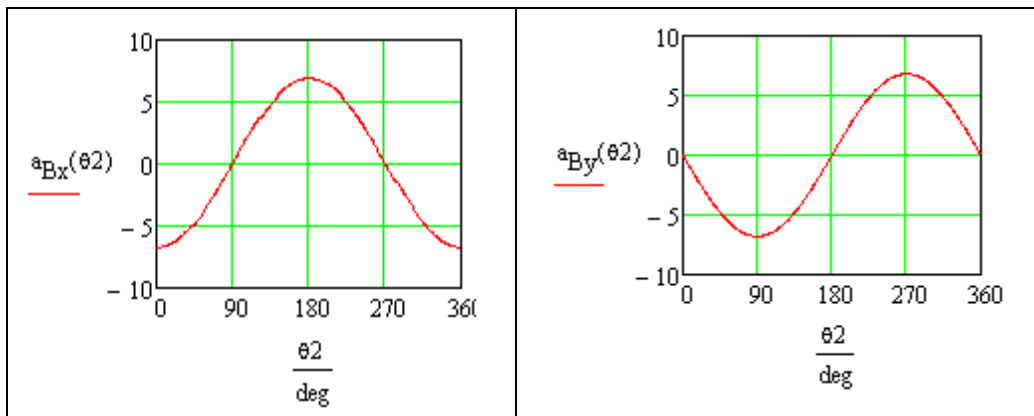


Figure 5: The position diagrams for the position equations of the point B in direction of x and y

$$a_{Bx}(\theta_2) := -r_2 \cdot \cos(\theta_2) \cdot \omega_2^2$$

$$a_{By}(\theta_2) := -r_2 \cdot \sin(\theta_2) \cdot \omega_2^2$$

$$a_B(\theta_2) := \sqrt{a_{Bx}(\theta_2)^2 + a_{By}(\theta_2)^2}$$

Angular velocity and acceleration for the point B :

$$\omega_4(\theta_2) := \frac{d}{d\theta_2} \theta_4(\theta_2) \cdot \omega_2$$

$$\varepsilon_4(\theta_2) := \frac{d^2}{d\theta_2^2} \theta_4(\theta_2) \cdot \omega_2^2$$

$$\omega_4(0.5) = -0.87$$

$$\varepsilon_4(0.5) = 0.445$$

1.3 The equations for position (displacement), velocity and acceleration of the point C

In the following is shown the displacement of point C , along with velocities and accelerations which are presented graphically by the diagrams via MathCad programme:

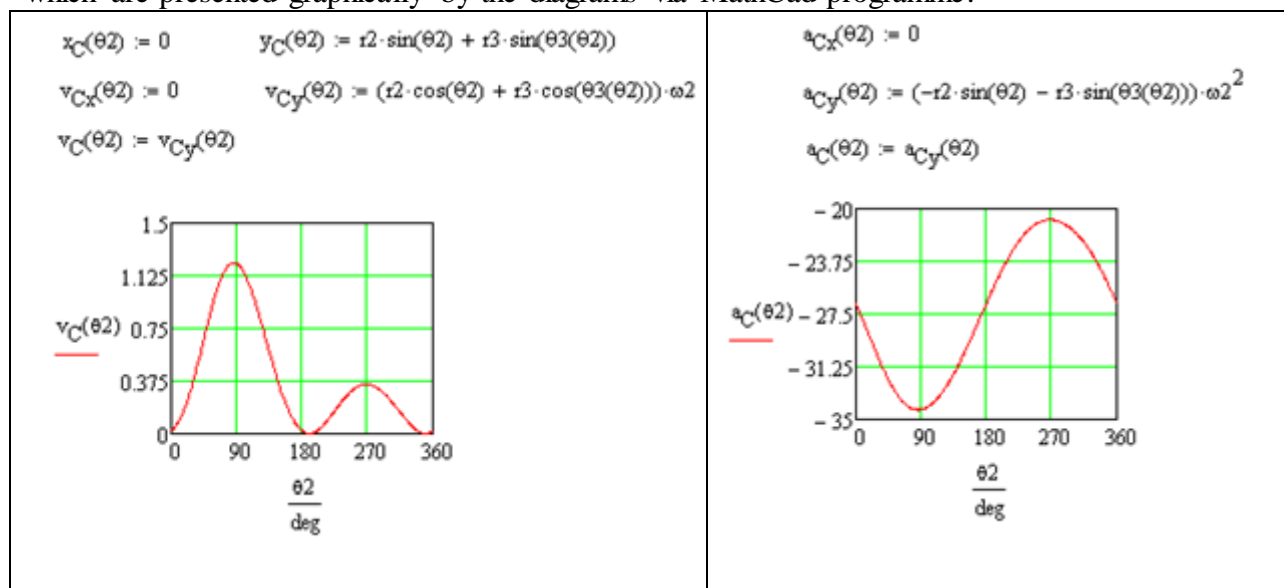


Figure 6 Velocities and acceleration of point C

1.4 The expression for the middle point of the bars AB, BC and CD

In the following are extracted the displacement for the middle position of the bars AB, BC and CD and other displacement which belong to these bars. Are also presented the velocities and accelerations of these bars and their diagrams separately:

$x_{C2}(\theta_2) := \frac{r2}{2} \cdot \cos(\theta_2)$	$y_{C2}(\theta_2) := \frac{r2}{2} \cdot \sin(\theta_2)$
$a_{xC2}(\theta_2) := \frac{-r2}{2} \cdot \cos(\theta_2) \cdot \omega_2^2$	$a_{yC2}(\theta_2) := \frac{-r2}{2} \cdot \sin(\theta_2) \cdot \omega_2^2$
BC	
$x_{C3}(\theta_2) := \frac{r3}{2} \cdot \cos(\theta_3(\theta_2))$	$y_{C3}(\theta_2) := \frac{r3}{2} \cdot \sin(\theta_3(\theta_2))$
$a_{xC3}(\theta_2) := \frac{-r3}{2} \cdot \cos(\theta_3(\theta_2)) \cdot \omega_2^2$	$a_{yC3}(\theta_2) := \frac{-r3}{2} \cdot \sin(\theta_3(\theta_2)) \cdot \omega_2^2$
CD	
$x_{C4}(\theta_2) := \frac{r4}{2} \cdot \cos(\theta_4(\theta_2))$	$y_{C4}(\theta_2) := \frac{r4}{2} \cdot \sin(\theta_4(\theta_2))$
$a_{xC4}(\theta_2) := \frac{-r4}{2} \cdot \cos(\theta_4(\theta_2)) \cdot \omega_2^2$	$a_{yC4}(\theta_2) := \frac{-r4}{2} \cdot \sin(\theta_4(\theta_2)) \cdot \omega_2^2$

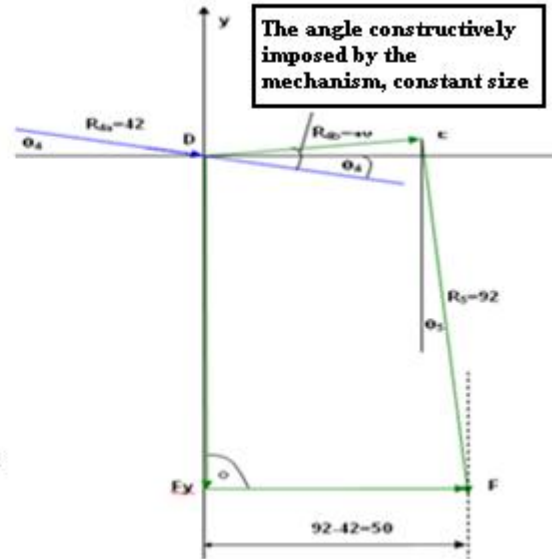
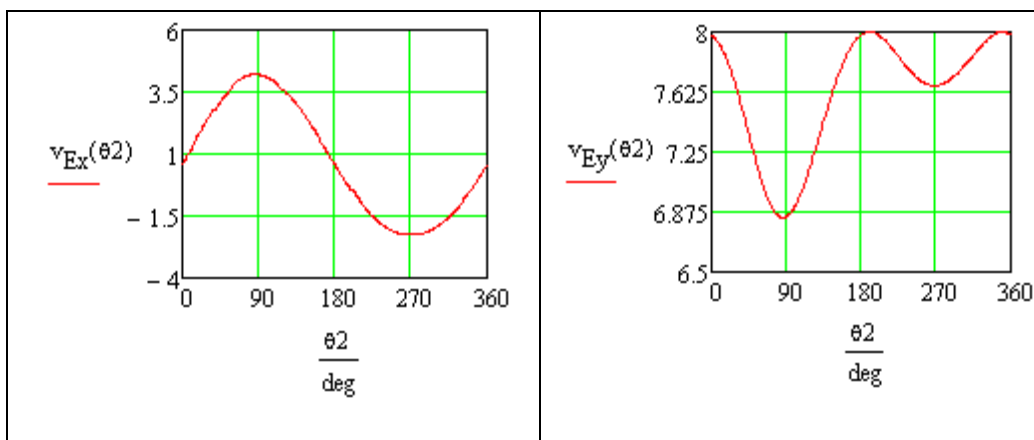


Figure 7: Crank mechanism

1.5 The equations for position (displacement), velocity and acceleration of the point E

For the point E of Crank Mechanism determined the displacement, and in the following pictures are shown velocities and accelerations in the direction x and y :

$x_E(\theta_2) := r4b \cdot \cos(\theta_4(\theta_2))$	$y_E(\theta_2) := r4b \cdot \sin(\theta_4(\theta_2))$
$v_{Ex}(\theta_2) := -r4b \cdot \sin(\theta_4(\theta_2)) \cdot \omega_2$	$v_{Ey}(\theta_2) := r4b \cdot \cos(\theta_4(\theta_2)) \cdot \omega_2$
$a_{Ex}(\theta_2) := -r4b \cdot \cos(\theta_4(\theta_2)) \cdot \omega_2^2$	$a_{Ey}(\theta_2) := -r4b \cdot \sin(\theta_4(\theta_2)) \cdot \omega_2^2$



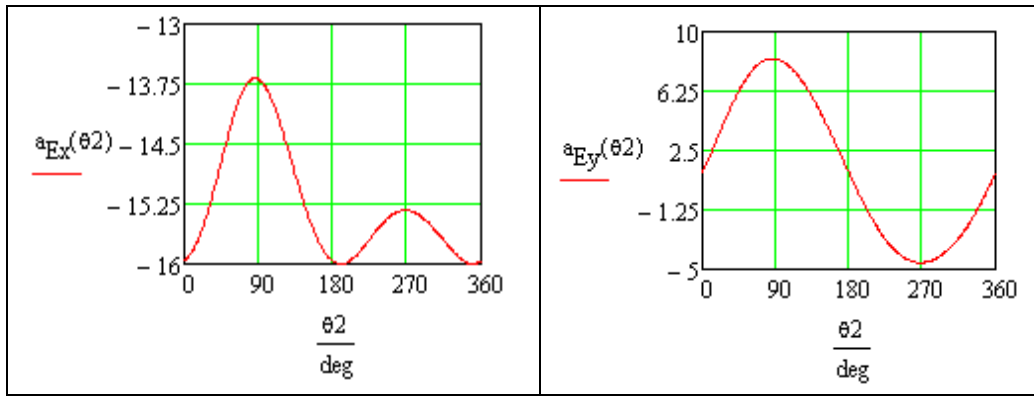


Figure 8: Diagrams of the velocities and acceleration for the E

1.6 Kinetostatic analysis of the six-bar linkage mechanism

For the kinetostatic part will be presented the kinetostatic analysis of six-bar linkage mechanism by MathCad software. Moments of inertia, masses of bodies are used in the following:

$$m_2 := 10 \quad m_3 := 15 \quad m_4 := 20 \quad m_5 := 15 \quad m_6 := 5 \quad g := 9.807$$

$$J_{C2} := \frac{1}{12} \cdot m_2 \cdot r_2^2 \quad J_{C3} := \frac{1}{12} \cdot m_3 \cdot r_3^2 \quad J_{C4} := \frac{1}{18} \cdot m_4 \cdot (2.4^2 + 1.2^2)$$

$$J_{C5} := \frac{1}{12} \cdot m_5 \cdot r_5^2 \quad \frac{1}{12} \cdot m_4 \cdot (2.4^2 + 12^2) = 249.6$$

Linkage I

The equilibrium conditions for the point A are equal to zero. Six linkages are used for the kinetostatic analysis. For the first linkage are given the following equilibrium conditions X_A , Y_A and X_B , Y_B and for the middle points of the linkage AB, point C_2 and for the body mass m_2 .

$$X_A := 0 \quad Y_A := 0 \quad X_B := 0 \quad Y_B := 0 \quad X_{C2} := 0 \quad Y_{C2} := 0 \quad X_D := 0 \quad Y_D := 0$$

$$X_E := 0 \quad Y_E := 0 \quad X_F := 0 \quad Y_F := 0 \quad NN := 0 \quad M_{tr} := 0$$

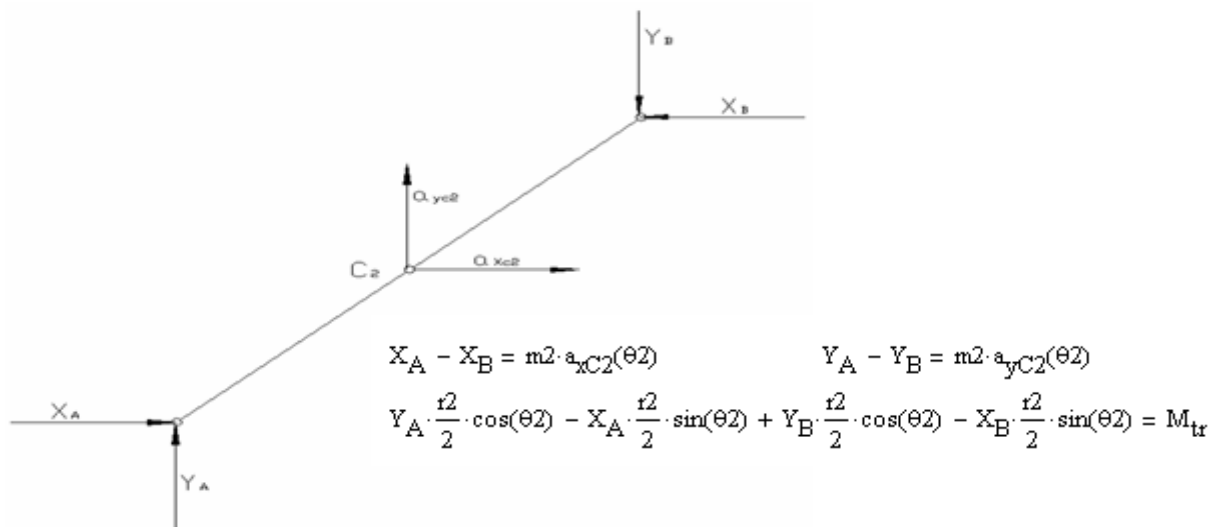
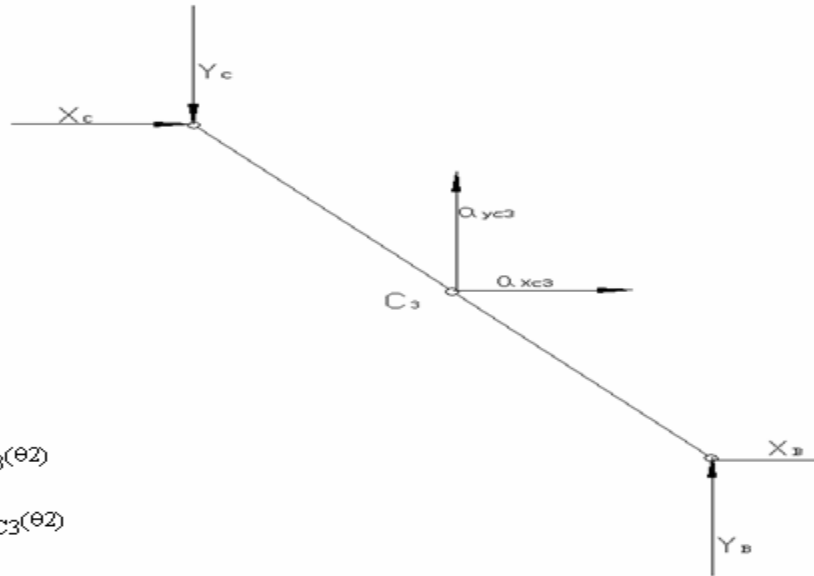


Figure 9: Linkage I-Equilibrium conditions for the point AB

Linkage II

Also, for the linkage II are written the equilibrium conditions BC, which are X_B, Y_B, X_C, Y_C , centre C_3 , body mass BC and moment of inertia for the point C_3 .



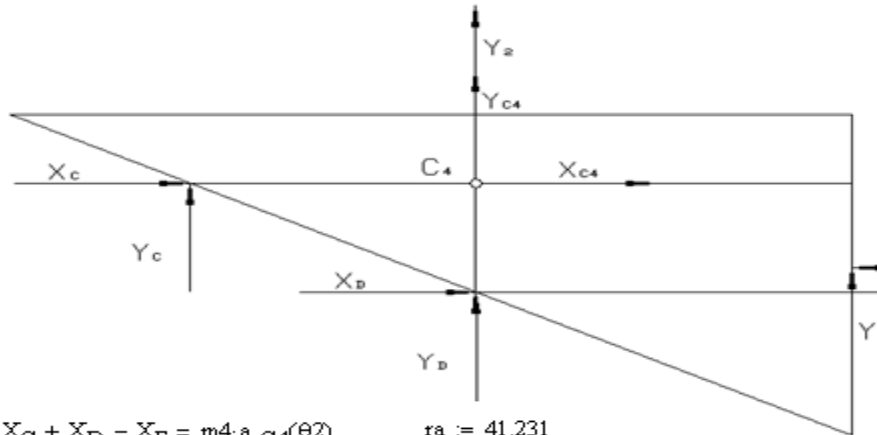
$$X_C + X_B = m_3 \cdot a_{xC3}(\theta_2)$$

$$Y_B - Y_C = m_3 \cdot a_{yC3}(\theta_2)$$

$$X_C \cdot \frac{r_3}{2} \cdot \sin(\theta_3(\theta_2)) - Y_C \cdot \frac{r_3}{2} \cdot \cos(\theta_3(\theta_2)) - Y_B \cdot \frac{r_3}{2} \cdot \cos(\theta_3(\theta_2)) - X_B \cdot \frac{r_3}{2} \cdot \sin(\theta_3(\theta_2)) = J_{C3} \cdot \varepsilon_3(\theta_2)$$

Figure 10: Linkage II – Equilibrium conditions for the part BC

Linkage III



$$X_C + X_D - X_E = m_4 \cdot a_{xC4}(\theta_2) \quad ra := 41.231$$

$$h := 0.8$$

$$Y_C + Y_D + Y_E = m_4 \cdot a_{yC4}(\theta_2) \quad EH := 1.2$$

$$-Y_C \cdot ra + Y_E \cdot r4b - X_E \cdot EH - Y_D \cdot rz(\theta_2) + X_D \cdot rm(\theta_2) = J_{C4} \cdot \varepsilon_4(\theta_2)$$

Figure 11: Linkage III of the points C, D, E and centre C_4

Halkka IV

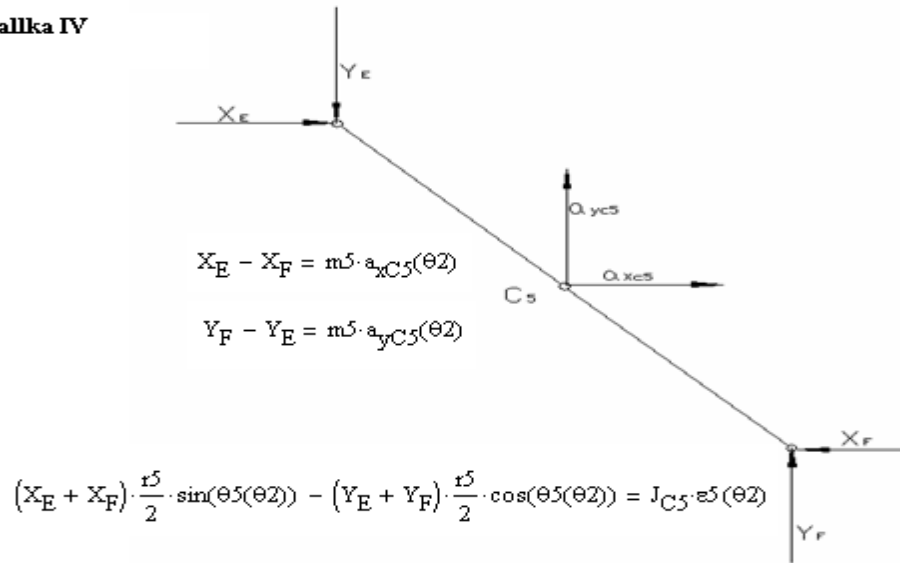


Figure 12: Linkage IV of the part EF in direction of x and y

Halkka V

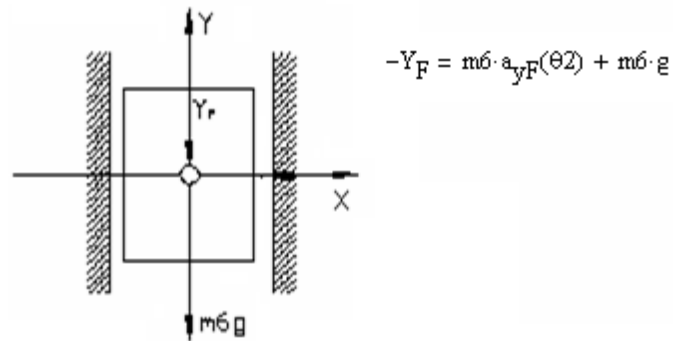


Figure 13: Linkage V of the point F in direction of x and y

All unknown parameters for the linkages in both directions x and y are calculated by MathCad. From point A to the point F and the Transmission moment M_{tr} :

$$Z_{gij}(\theta_2) := \text{Find}(X_A, Y_A, X_B, Y_B, X_C, Y_C, X_D, Y_D, X_E, Y_E, X_F, Y_F, M_{tr})$$

$$X_{A_i}(\theta_2) := Z_{gij}(\theta_2)_0 \quad Y_{A_i}(\theta_2) := Z_{gij}(\theta_2)_1$$

$$X_{B_i}(\theta_2) := Z_{gij}(\theta_2)_2 \quad Y_{B_i}(\theta_2) := Z_{gij}(\theta_2)_3$$

$$X_{C_i}(\theta_2) := Z_{gij}(\theta_2)_4 \quad Y_{C_i}(\theta_2) := Z_{gij}(\theta_2)_5$$

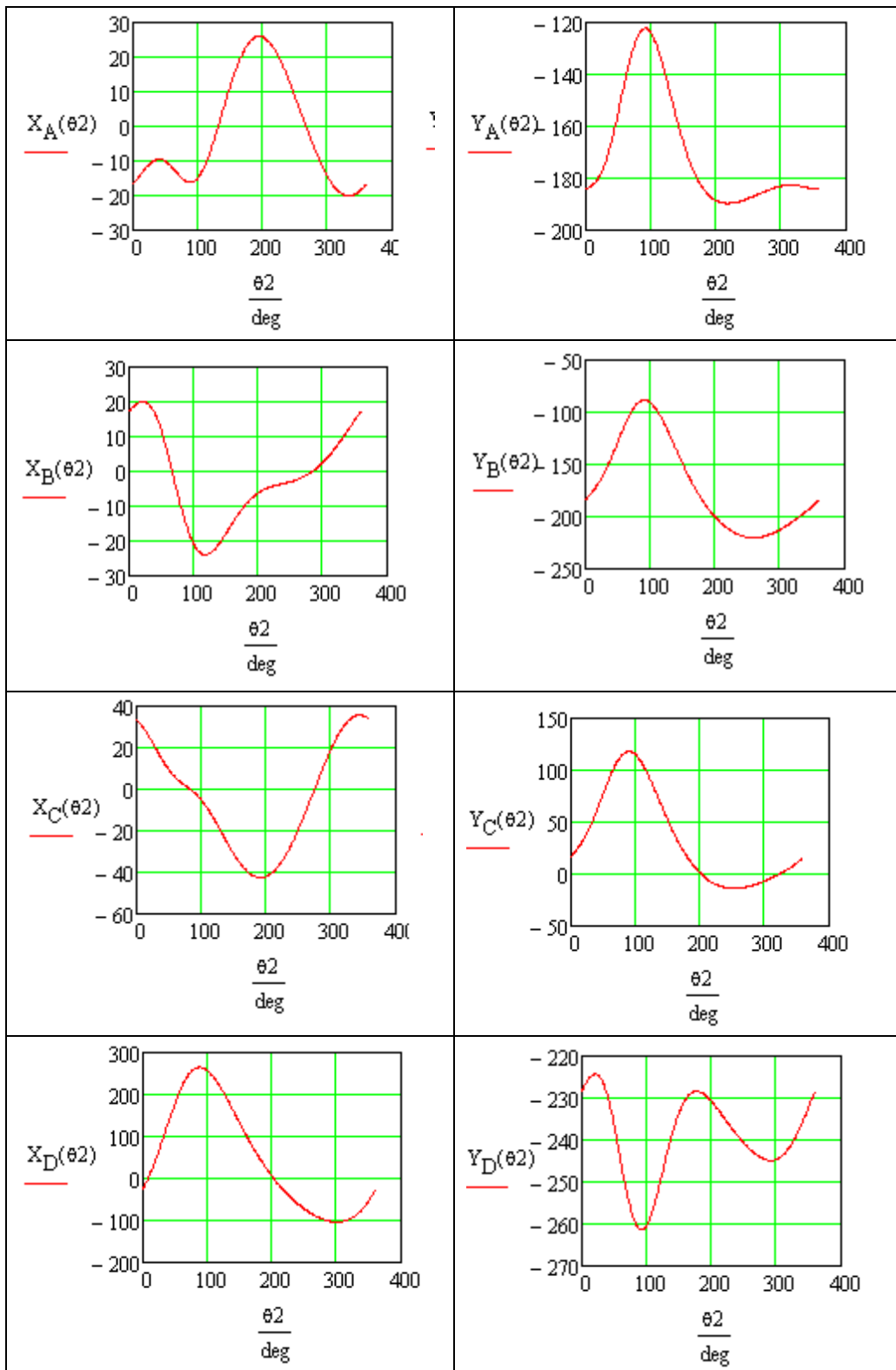
$$X_{D_i}(\theta_2) := Z_{gij}(\theta_2)_6 \quad Y_{D_i}(\theta_2) := Z_{gij}(\theta_2)_7$$

$$X_{F_i}(\theta_2) := Z_{gij}(\theta_2)_8 \quad Y_{F_i}(\theta_2) := Z_{gij}(\theta_2)_9$$

$$X_{F_j}(\theta_2) := Z_{gij}(\theta_2)_{10} \quad Y_{F_j}(\theta_2) := Z_{gij}(\theta_2)_{11}$$

$$M_{tr_i}(\theta_2) := Z_{gij}(\theta_2)_{12}$$

In the following diagrams are given the values of all unknown parameters for the positions (0; 0.5; 1) by MathCad software. This it was really a tough work since the working process by MathCad was very slow especially to follow the procedure which is given by kinetostatic analysis.



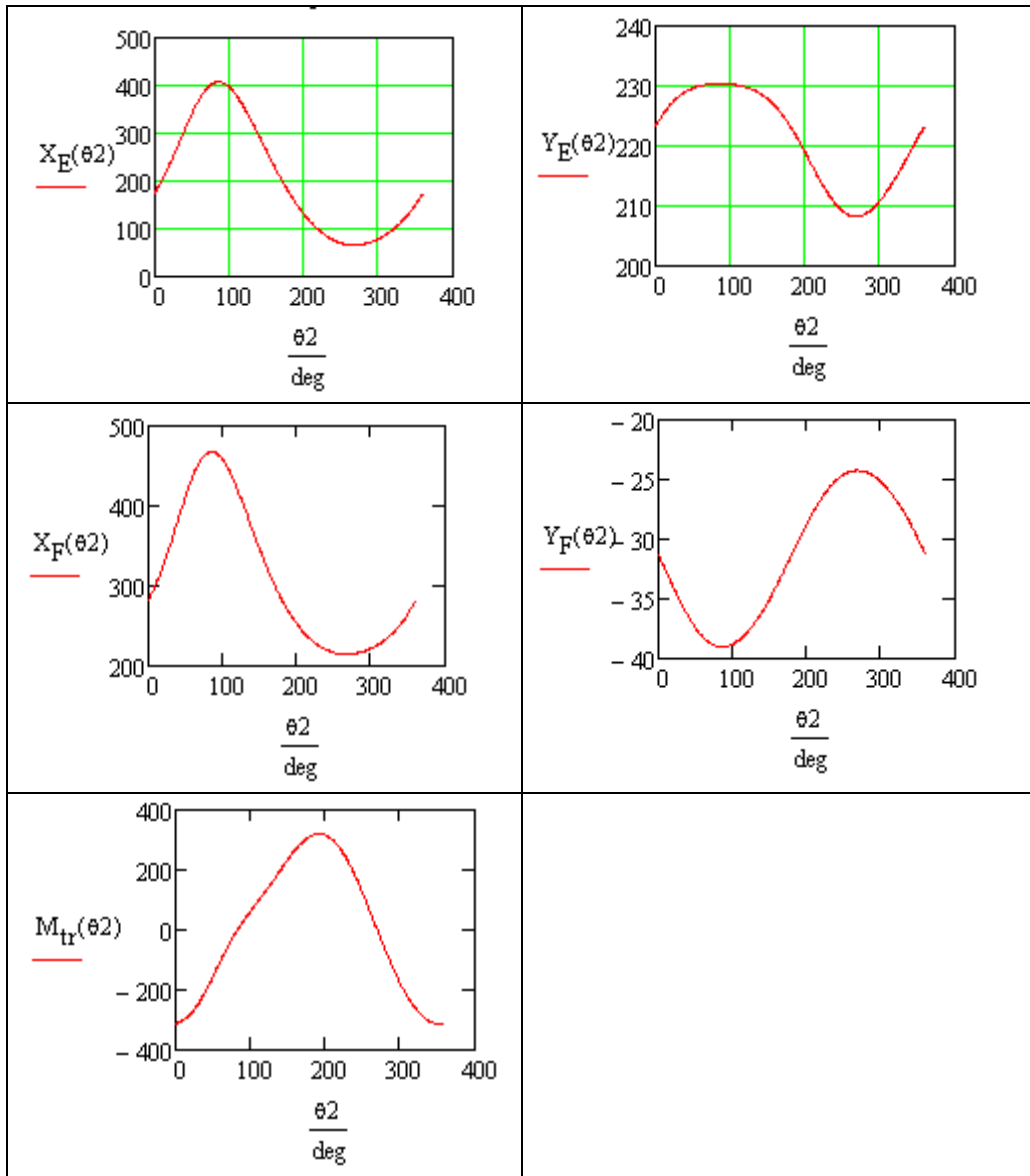


Figure 14: Diagrams of kinestatic analysis of the given mechanism for the points A, B, C, D, E, F and transmission moment M_{tr}

2. Simulation for six-bar mechanism by Working Model software

In the second part of this paper is carried out the simulation for all points A, B, C, D, E, F and M_{tr} (time dependent) of the six bar linkage mechanism by Working Model, which is shown in the following.

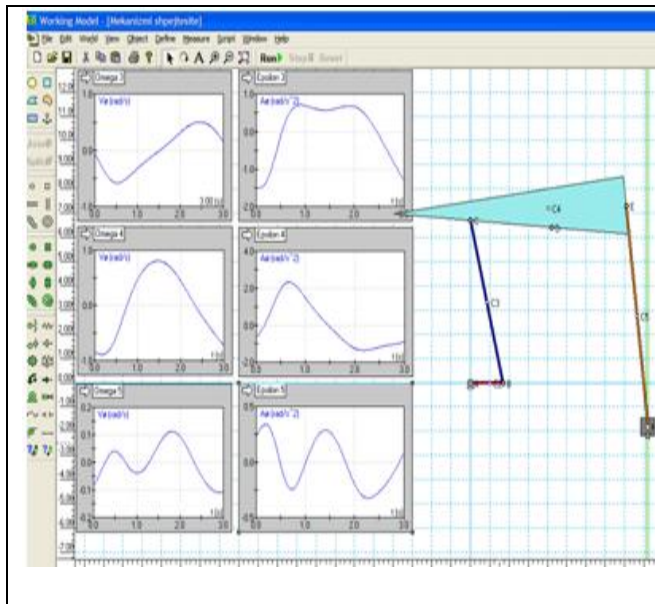


Figure 15: Diagrams in Working Model for angular velocity and acceleration 3, 4 and 5

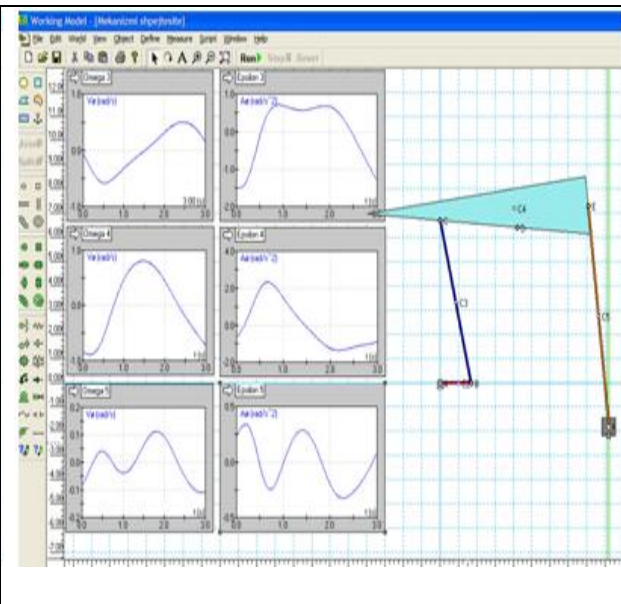


Figure 16: Diagrams in Working Model for equilibrium points A, B, C, D, E, F and transmission moment (M_{tr}) in direction x

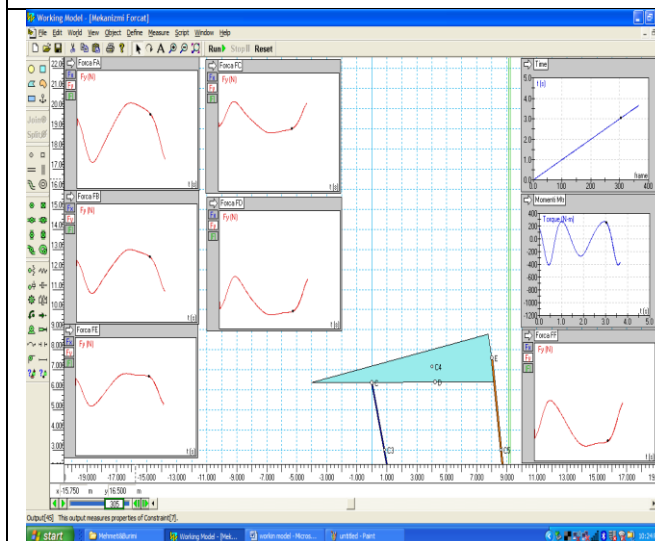


Figure 17: Diagrams in Working Model for equilibrium points A, B, C, D, E, F and transmission moment (M_{tr}) in direction y

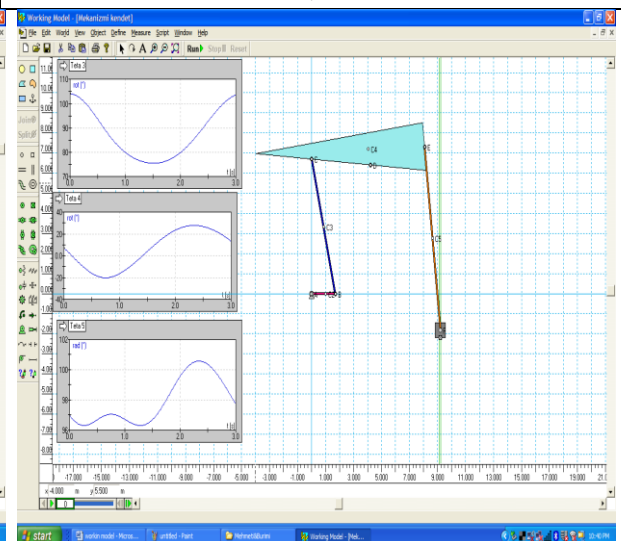


Figure 18: Diagrams for θ_3 , θ_4 and θ_5 (time dependent), under command RUN

1. Conclusions and recommendations

- By application of the software's MathCad or MatLab we have reached to carry out the full analysis of this paper. In this paper are made the calculations of all positions (displacement) for the whole mechanism, and also are determined the plans for velocities and accelerations for each point.
- However, in this paper are shown the outline planes of the mechanism same as the diagrams for each linkage through MathCad and Working Model software,

- Six-bar linkage mechanism diagrams which are derived by Working Model are almost similar to the diagrams derived by MathCad, same as the derived results,
- Through Working Model software, are derived the results of reactions from the equilibrium conditions of six bar linkage mechanism, angular velocities and accelerations for the points 3, 4 and 5, for the angles θ_3 , θ_4 and θ_5 in time domain through the command Run,
- As the general conclusion; the results derived by both software, same as for their diagrams, for all points of the six bar linkage mechanism are within the reasonable boundaries. Our expectations are that similar results will be derived by application of MatLab software.

4. References

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