Computer Aided Analysis Of Kinematics And Kinetostatics Of Six-Bar Linkage Mechanism Through The Contour Method

Qelaj Mehmet
Ministry of Economic Development, mehmet.qelaj@gmail.com

Likaj Ramë
Ministry of Economic Development, ramelikaj@yahoo.com

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Qelaj Mehmet
Ministry of Economic Development
Mother Theresa Street - No. 36 - 10000 Pristina

Likaj Ramë
Mechanical Engineering Faculty
Technical Faculty, n.n - 10000 Pristina
mehmet.qelaj@gmail.com ramelikaj@yahoo.com

Abstract

In this paper is presented a six-bar linkage mechanism of the pump for oil extrusion. In this mechanism are introduced even higher kinematic pairs. Dimensions and other incoming links will be adopted as necessary. For the six-bar linkage mechanism will be carried out the kinematic analysis and for all linkages will be presented the displacement, velocity and acceleration. The analysis will be performed by MathCad software, while kinetostatic analysis will be carried out using Contour Method, comparing results of two different software's MathCAD and Working Model. The simulation parameters will be computed for all points of the contours of mechanism.

Keywords: mechanism, contour, displacement, speed, acceleration

This paper has been realised with two application software's: MathCad and Working Model. In this paper is presented a four-bar linkage ABCD and a simple crank mechanism DEF as shown in the figure below. In this mechanism firstly is determined DF_y and θ_5. Derivative DF_y represents the speed of the slider F. From the given picture $\theta_2 = 0,1,...,360^o$ should be determined. The masses are adopted, since the moments of inertia need to calculate. The kinematic part of the paper will be completed by finding the velocities and accelerations of each point A, B, C, D, E, F for the centres from C_1 to C_6. In this way are determined the angular accelerations and velocities of the linkages 2, 3, 4 and 5. Whereas for the kinetostatic part will be determined the reaction forces of the points: X_A, Y_A, X_B, Y_B, X_C, Y_C, X_D, Y_D, X_E, Y_E, X_F, Y_F, $N = X_{F6}$ and M_{tr} which are acting on the leading link 2. In the picture are shown 5 bodies: AB, BC, right triangle, rod EF, slider bar F.

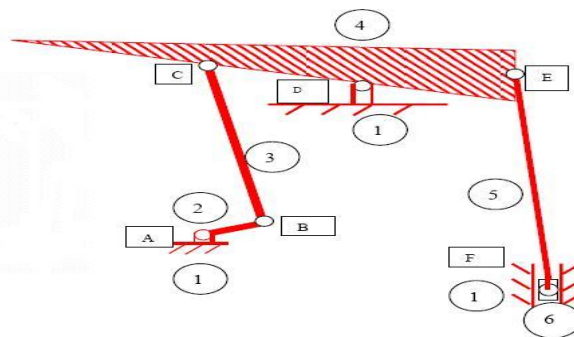**Figure 1: Six bar-linkage mechanism** $ABCDEF$

Given data:

$$\theta_2 := 0,5 deg .. 360 deg \qquad \omega_2 := 2$$

$$r_2 := 1.7 \qquad r_3 := 6.9 \qquad r_{4a} := 4.2 \qquad r_{4b} := 4.0 \qquad r_5 := 9.2 \qquad r_D := \sqrt{4.2^2 + 6.4^2}$$

$$\alpha := \mathrm{acos}\left(\frac{4.2}{\sqrt{4.2^2 + 6.4^2}}\right) \qquad\qquad \frac{\alpha}{\deg} = 56.725$$

Determination of angle α of r_D with x_1:

Linkage I

$$\vec{r_2} + \vec{r_3} + \vec{r_{4a}} = \vec{r_D}$$

From then designed outline at x and y we have the following initial conditions:

Given

$$r_2 \cdot \cos(\theta_2) + r_3 \cdot \cos(\theta_3) + r_{4a} \cdot \cos(\theta_4) = r_D \cdot \cos(\alpha)$$
$$r_2 \cdot \sin(\theta_2) + r_3 \cdot \sin(\theta_3) + r_{4a} \cdot \sin(\theta_4) = r_D \cdot \sin(\alpha)$$
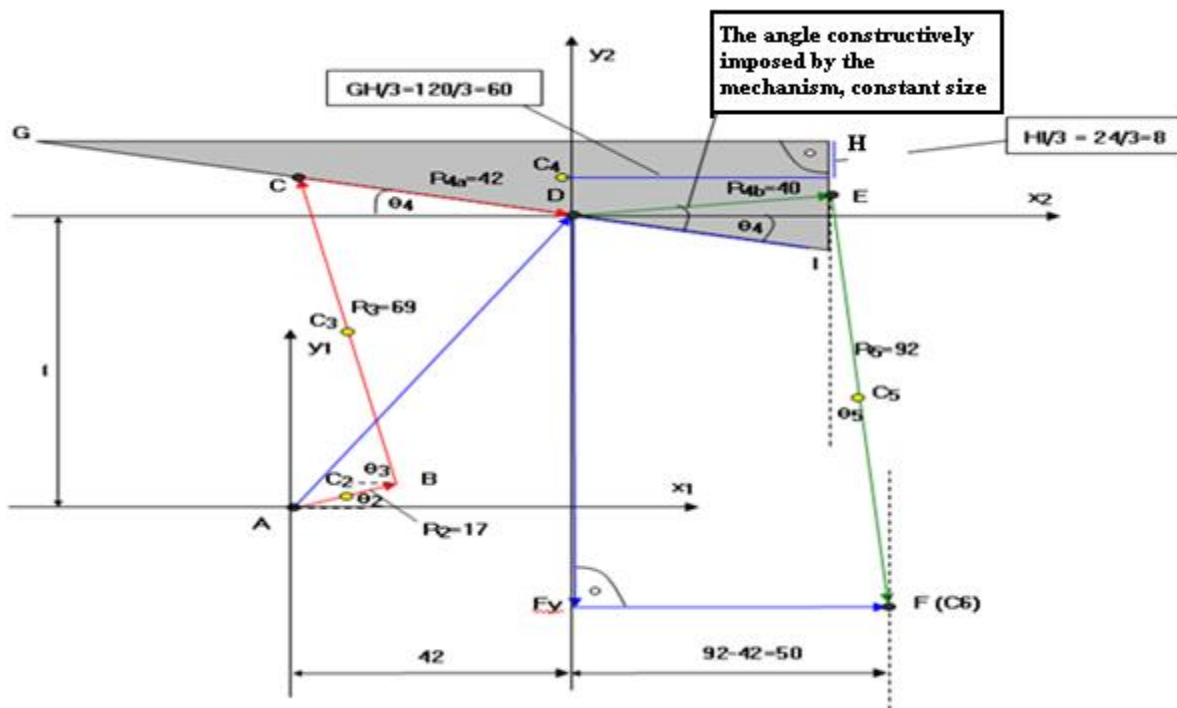

Figure 2: Four-bar linkage $ABCD$ and crank mechanism DEF

As shown in the Figure 2, there are 14 unknown sizes, 12 equations for the first four bodies, $dy(x, y)$ for the slider 14 equations.

1.1 The analysis of the positions, velocities and accelerations of six-bar linkage mechanism

In the figure 3 is presented the velocity plan for the points A, B, C and D. Also are presented angular velocities and angular accelerations for the points A, B and C. After finding the velocities and accelerations of these points, the displacements, velocities and accelerations of these points with angles θ_3 and θ_4 in function of (θ_2) are shown graphically. Outline is shown in this form:
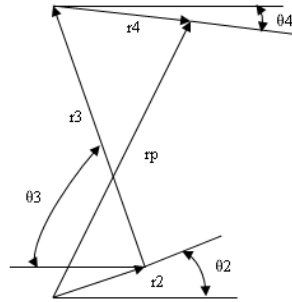
Figure 3: Velocity plan for the points A, B and C

Given $\qquad\qquad$ θ3 := 180·deg $\qquad\qquad$ θ4 := 90·deg

Ekuacioni vektorial për konturin I:

$$
\begin{pmatrix} r2\cdot\cos(\theta2) \\ r2\cdot\sin(\theta2) \\ 0 \end{pmatrix} + \begin{pmatrix} r3\cdot\cos(\theta3) \\ r3\cdot\sin(\theta3) \\ 0 \end{pmatrix} + \begin{pmatrix} r4\cdot\cos(\theta4) \\ r4\cdot\sin(\theta4) \\ 0 \end{pmatrix} = \begin{pmatrix} rp\cdot\cos(\alpha) \\ rp\cdot\sin(\alpha) \\ 0 \end{pmatrix}
$$
....(1)

Zgj(θ2) := Find(θ3,θ4)

$\qquad$ θ3(θ2) := Zgj(θ2)$_0$

$\qquad$ θ4(θ2) := Zgj(θ2)$_1$ $\qquad\qquad$ $\beta := \operatorname{asin}\left(\dfrac{2.4}{\sqrt{1.2^2 + 2.4^2}}\right)$ $\qquad$ $\dfrac{\beta}{\deg} - \dfrac{-\theta4(0)}{\deg} = 59.478$

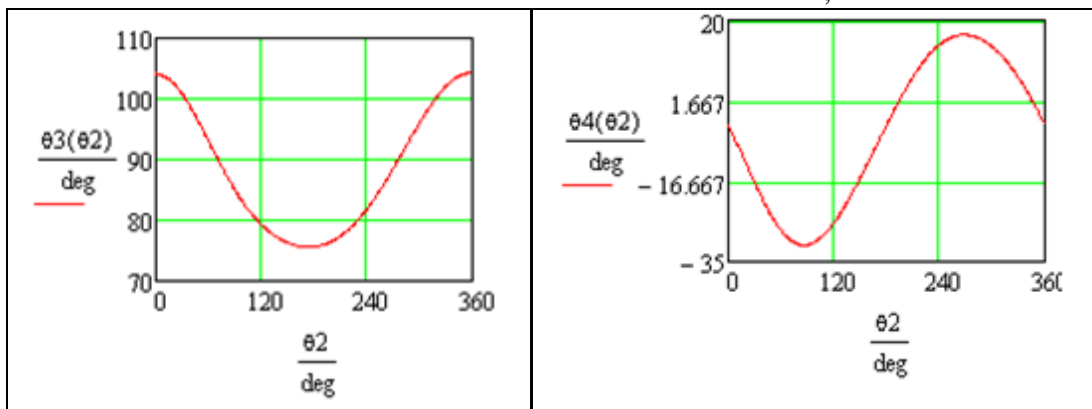$\qquad \dfrac{\theta3(0)}{\deg} = 104.177$ $\qquad\qquad\qquad \dfrac{\theta4(0)}{\deg} = -3.957$

$\qquad \omega_3(\theta2) := \dfrac{d}{d\theta2}\theta3(\theta2)\cdot\omega2$ $\qquad\qquad \omega_4(\theta2) := \dfrac{d}{d\theta2}\theta4(\theta2)\cdot\omega2$

$\qquad \varepsilon_3(\theta2) := \dfrac{d^2}{d\theta2^2}\theta3(\theta2)\cdot\omega2^2$ $\qquad\qquad \varepsilon_4(\theta2) := \dfrac{d^2}{d\theta2^2}\theta4(\theta2)\cdot\omega2^2$

After calculation of the values by MathCad software, in the following are shown graphically the values of positions, velocities and accelerations respectively for θ_3, ω_3, ε_3 and θ_4, ω_4, ε_4:
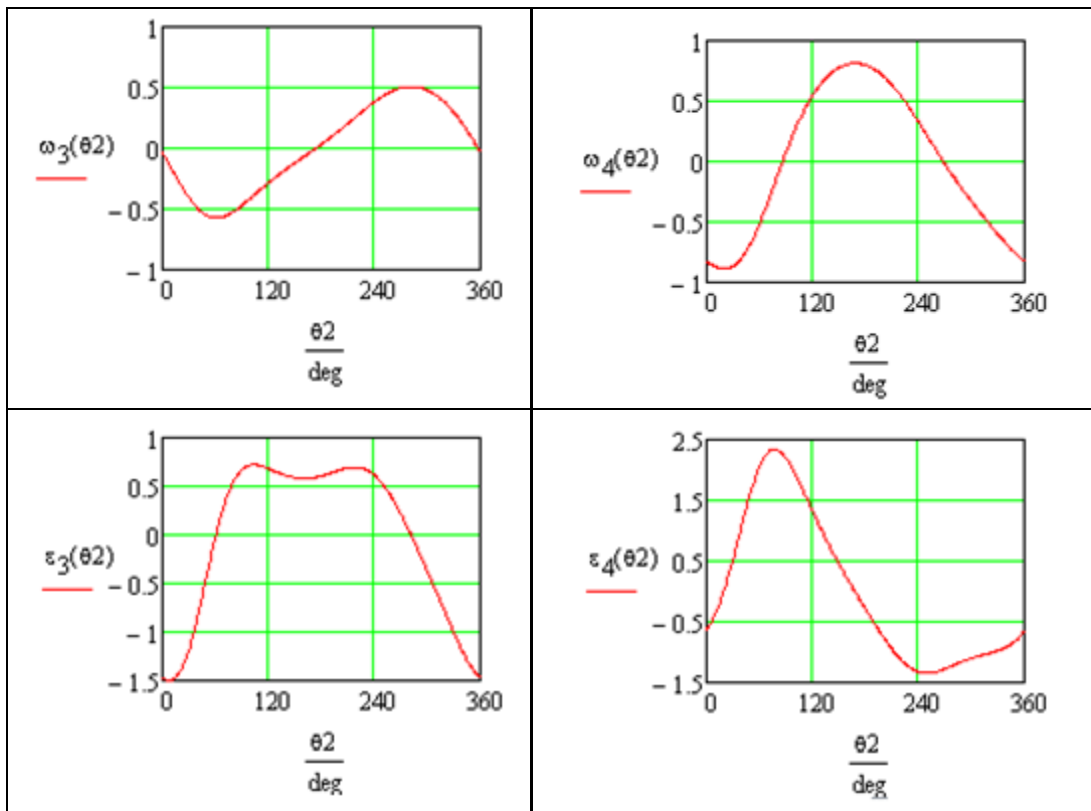
Figure 4: Diagrams for positions, velocities and accelerations

1.2 Position of the point B

The position of the point B is calculated in direction of x and y:
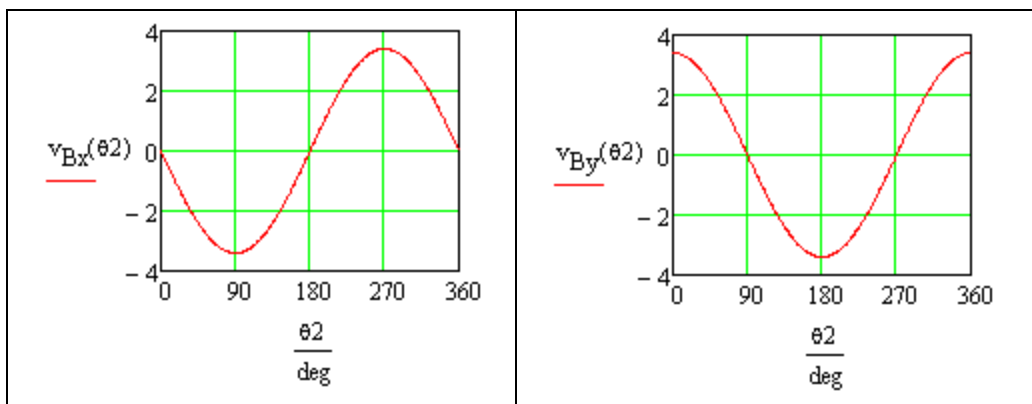
$$x_B(\theta2) := r2 \cdot \cos(\theta2) \qquad\qquad y_B(\theta2) := r2 \cdot \sin(\theta2)$$

$$v_{Bx}(\theta2) := -r2 \cdot \sin(\theta2) \cdot \omega2 \qquad\qquad v_{By}(\theta2) := r2 \cdot \cos(\theta2) \cdot \omega2$$

$$v_B(\theta2) := \sqrt{v_{Bx}(\theta2)^2 + v_{By}(\theta2)^2} \qquad \sqrt{9.2^2 + 4.0^2} = 10.032$$
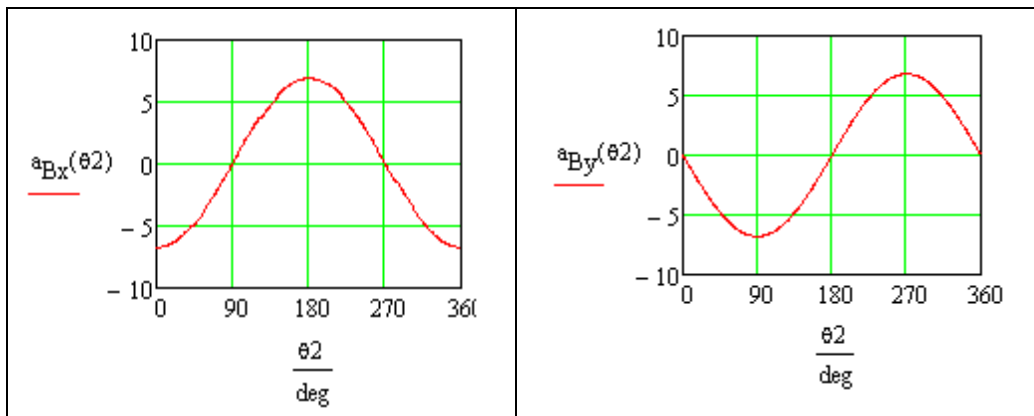
Figure 5: The position diagrams for the position equations of the point B in direction of x and y

$$a_{Bx}(\theta 2) := -r2 \cdot \cos(\theta 2) \cdot \omega 2^2$$

$$a_{By}(\theta 2) := -r2 \cdot \sin(\theta 2) \cdot \omega 2^2$$

$$a_B(\theta 2) := \sqrt{a_{Bx}(\theta 2)^2 + a_{By}(\theta 2)^2}$$

Angular velocity and acceleration for the point B :

$$\omega_4(\theta 2) := \frac{d}{d\theta 2} \theta 4(\theta 2) \cdot \omega 2 \qquad\qquad \varepsilon_4(\theta 2) := \frac{d^2}{d\theta 2^2} \theta 4(\theta 2) \cdot \omega 2^2$$

$$\omega_4(0.5) = -0.87 \qquad\qquad \varepsilon_4(0.5) = 0.445$$

1.3 The equations for position (displacement), velocity and acceleration of the point C

In the following is shown the displacement of point C, along with velocities and accelerations which are presented graphically by the diagrams via MathCad programme:
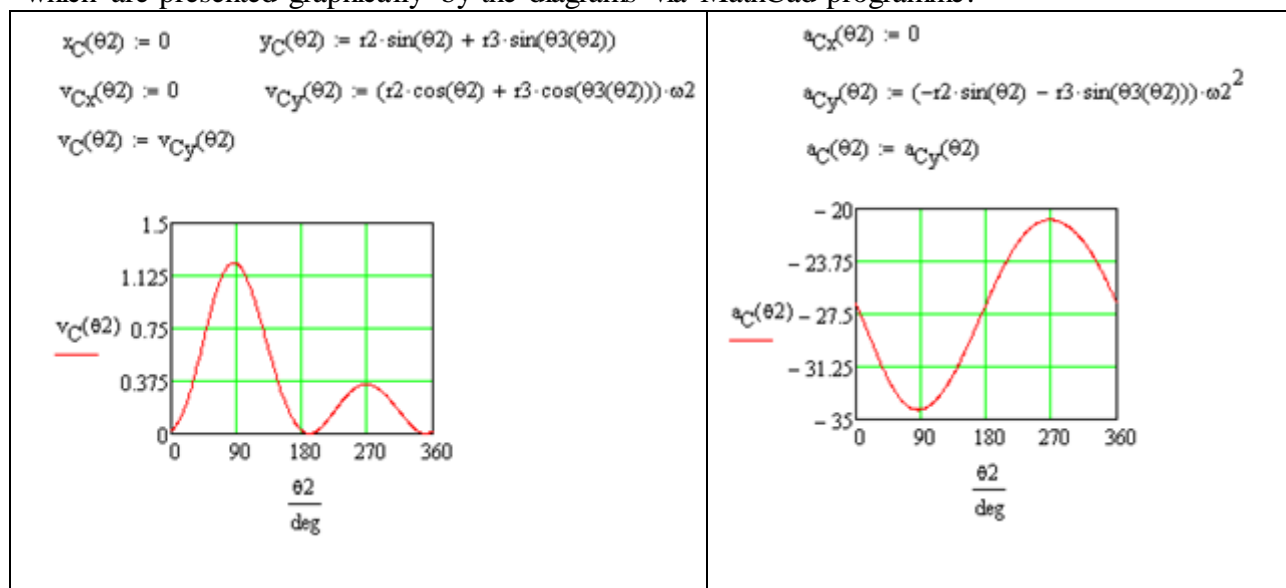

$$x_C(\theta 2) := 0 \qquad\qquad y_C(\theta 2) := r2 \cdot \sin(\theta 2) + r3 \cdot \sin(\theta 3(\theta 2))$$

$$v_{Cx}(\theta 2) := 0 \qquad\qquad v_{Cy}(\theta 2) := (r2 \cdot \cos(\theta 2) + r3 \cdot \cos(\theta 3(\theta 2))) \cdot \omega 2$$

$$v_C(\theta 2) := v_{Cy}(\theta 2)$$

$$a_{Cx}(\theta 2) := 0$$

$$a_{Cy}(\theta 2) := (-r2 \cdot \sin(\theta 2) - r3 \cdot \sin(\theta 3(\theta 2))) \cdot \omega 2^2$$

$$a_C(\theta 2) := a_{Cy}(\theta 2)$$

Figure 6 Velocities and acceleration of point C

1.4 The expression for the middle point of the bars AB, BC and CD

In the following are extracted the displacement for the middle position of the bars AB, BC and CD and other displacement which belong to these bars. Are also presented the velocities and accelerations of these bars and their diagrams separately:

$$x_{C2}(\theta2) := \frac{r2}{2}\cdot\cos(\theta2) \qquad y_{C2}(\theta2) := \frac{r2}{2}\cdot\sin(\theta2)$$

$$a_{xC2}(\theta2) := \frac{-r2}{2}\cdot\cos(\theta2)\cdot\omega2^2 \qquad a_{yC2}(\theta2) := \frac{-r2}{2}\cdot\sin(\theta2)\cdot\omega2^2$$

BC

$$x_{C3}(\theta2) := \frac{r3}{2}\cdot\cos(\theta3(\theta2)) \qquad y_{C3}(\theta2) := \frac{r3}{2}\cdot\sin(\theta3(\theta2))$$

$$a_{xC3}(\theta2) := \frac{-r3}{2}\cdot\cos(\theta3(\theta2))\cdot\omega2^2 \qquad a_{yC3}(\theta2) := \frac{-r3}{2}\cdot\sin(\theta3(\theta2))\cdot\omega2^2$$

CD

$$x_{C4}(\theta2) := \frac{r4}{2}\cdot\cos(\theta4(\theta2)) \qquad y_{C4}(\theta2) := \frac{r4}{2}\cdot\sin(\theta4(\theta2))$$

$$a_{xC4}(\theta2) := \frac{-r4}{2}\cdot\cos(\theta4(\theta2))\cdot\omega2^2 \qquad a_{yC4}(\theta2) := \frac{-r4}{2}\cdot\sin(\theta4(\theta2))\cdot\omega2^2$$
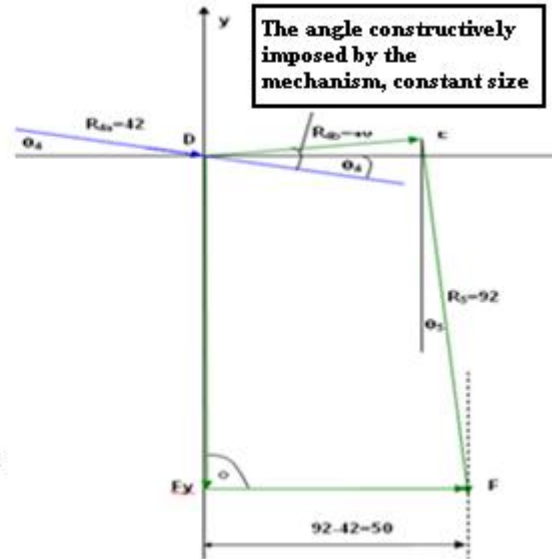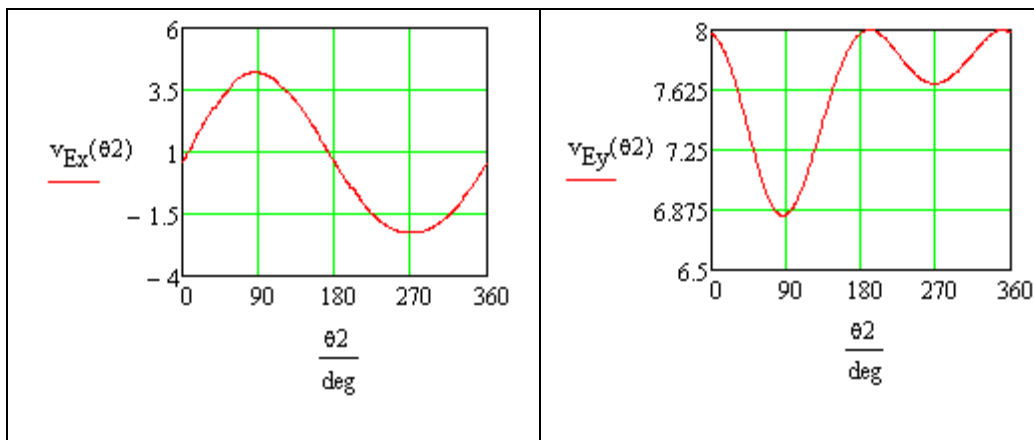

Figure 7: Crank mechanism

1.5 The equations for position (displacement), velocity and acceleration of the point E

For the point E of Crank Mechanism determined the displacement, and in the following pictures are shown velocities and accelerations in the direction x and y:

$$x_E(\theta2) := r4b\cdot\cos(\theta4(\theta2)) \qquad y_E(\theta2) := r4b\cdot\sin(\theta4(\theta2))$$

$$v_{Ex}(\theta2) := -r4b\cdot\sin(\theta4(\theta2))\cdot\omega2 \qquad v_{Ey}(\theta2) := r4b\cdot\cos(\theta4(\theta2))\cdot\omega2$$

$$a_{Ex}(\theta2) := -r4b\cdot\cos(\theta4(\theta2))\cdot\omega2^2 \qquad a_{Ey}(\theta2) := -r4b\cdot\sin(\theta4(\theta2))\cdot\omega2^2$$
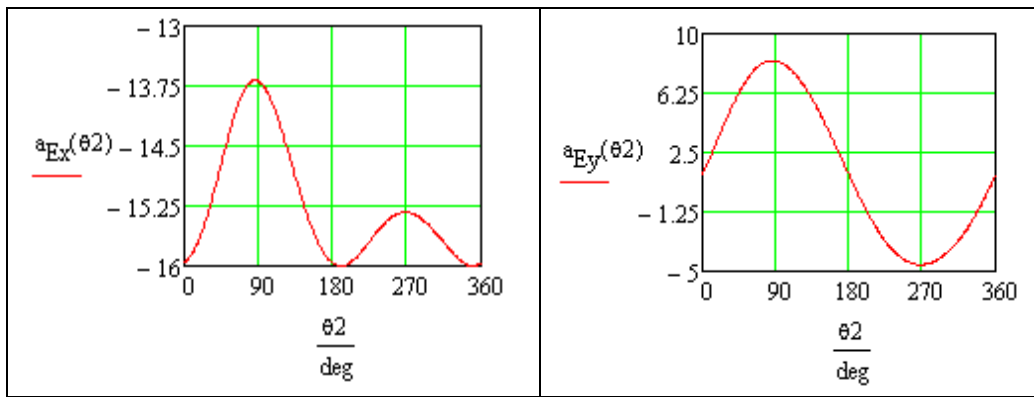
1.6 Kinetostatic analysis of the six-bar linkage mechanism

For the kinetostatic part will be presented the kinetostatic analysis of six-bar linkage mechanism by MathCad software. Moments of inertia, masses of bodies are used in the following:

$$m2 := 10 \qquad m3 := 15 \qquad m4 := 20 \qquad m5 := 15 \quad m6 := 5 \qquad g := 9.807$$

$$J_{C2} := \frac{1}{12} \cdot m2 \cdot r2^2 \qquad J_{C3} := \frac{1}{12} \cdot m3 \cdot r3^2 \qquad J_{C4} := \frac{1}{18} m4 \left(2.4^2 + 1.2^2\right)$$

$$J_{C5} := \frac{1}{12} \cdot m5 \cdot r5^2 \qquad \frac{1}{12} m4 \left(2.4^2 + 12^2\right) = 249.6$$

Linkage I

The equilibrium conditions for the point A are equal to zero. Six linkages are used for the kinetostatic analysis. For the first linkage are given the following equilibrium conditions X_A, Y_A and X_B, Y_B and for the middle points of the linkage AB, point C_2 and for the body mass m_2.

$$X_A := 0 \quad Y_A := 0 \quad X_B := 0 \quad Y_B := 0 \qquad X_C := 0 \qquad Y_C := 0 \qquad X_D := 0 \qquad Y_D := 0$$

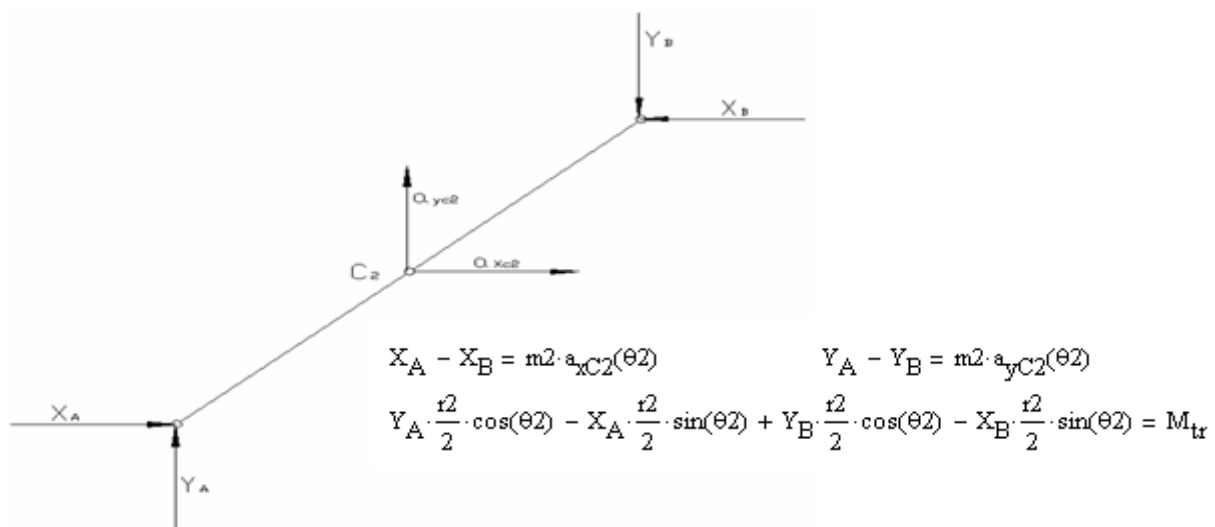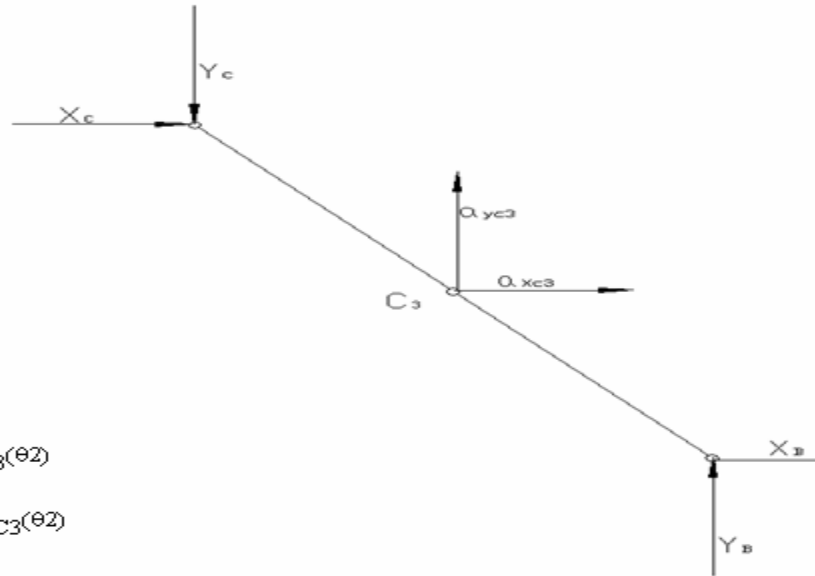$$X_E := 0 \quad Y_E := 0 \quad X_F := 0 \quad Y_F := 0 \qquad NN := 0 \qquad M_{tr} := 0$$

$$X_A - X_B = m2 \cdot a_{xC2}(\theta2) \qquad\qquad Y_A - Y_B = m2 \cdot a_{yC2}(\theta2)$$

$$Y_A \cdot \frac{r2}{2} \cdot \cos(\theta2) - X_A \cdot \frac{r2}{2} \cdot \sin(\theta2) + Y_B \cdot \frac{r2}{2} \cdot \cos(\theta2) - X_B \cdot \frac{r2}{2} \cdot \sin(\theta2) = M_{tr}$$

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Linkage II

Also, for the linkage II are written the equilibrium conditions BC, which are X_B, Y_B, X_C, Y_C, centre C_3, body mass BC and moment of inertia for the point C_3.
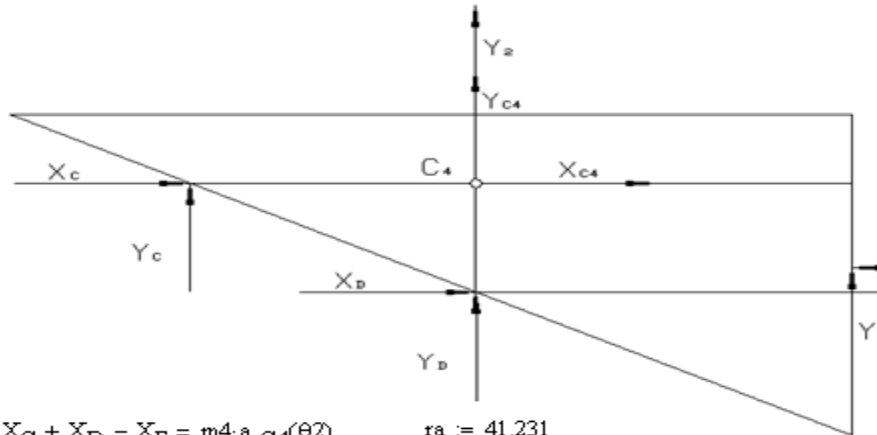
$$X_C + X_B = m3 \cdot a_{xC3}(\theta 2)$$

$$Y_B - Y_C = m3 \cdot a_{yC3}(\theta 2)$$

$$X_C \cdot \frac{r3}{2} \cdot \sin(\theta 3(\theta 2)) - Y_C \cdot \frac{r3}{2} \cdot \cos(\theta 3(\theta 2)) - Y_B \cdot \frac{r3}{2} \cdot \cos(\theta 3(\theta 2)) - X_B \cdot \frac{r3}{2} \cdot \sin(\theta 3(\theta 2)) = J_{C3} \cdot \varepsilon_3(\theta 2)$$

Figure 10: Linkage II – Equilibrium conditions for the part BC

Linkage III

$$X_C + X_D - X_E = m4 \cdot a_{xC4}(\theta 2)$$

$$Y_C + Y_D + Y_E = m4 \cdot a_{yC4}(\theta 2)$$

$$-Y_C \cdot ra + Y_E \cdot r4b - X_E \cdot EH - Y_D \cdot rz(\theta 2) + X_D \cdot rm(\theta 2) = J_{C4} \cdot \varepsilon_4(\theta 2)$$

$$ra := 41.231$$
$$h := 0.8$$
$$EH := 1.2$$

Figure 11: Linkage III of the points C, D, E and centre C_4

Hallka IV

$$X_E - X_F = m5 \cdot a_{xC5}(\theta 2)$$

$$Y_F - Y_E = m5 \cdot a_{yC5}(\theta 2)$$

$$\left(X_E + X_F\right) \cdot \frac{r5}{2} \cdot \sin(\theta 5(\theta 2)) - \left(Y_E + Y_F\right) \cdot \frac{r5}{2} \cdot \cos(\theta 5(\theta 2)) = J_{C5} \cdot \varepsilon 5(\theta 2)$$
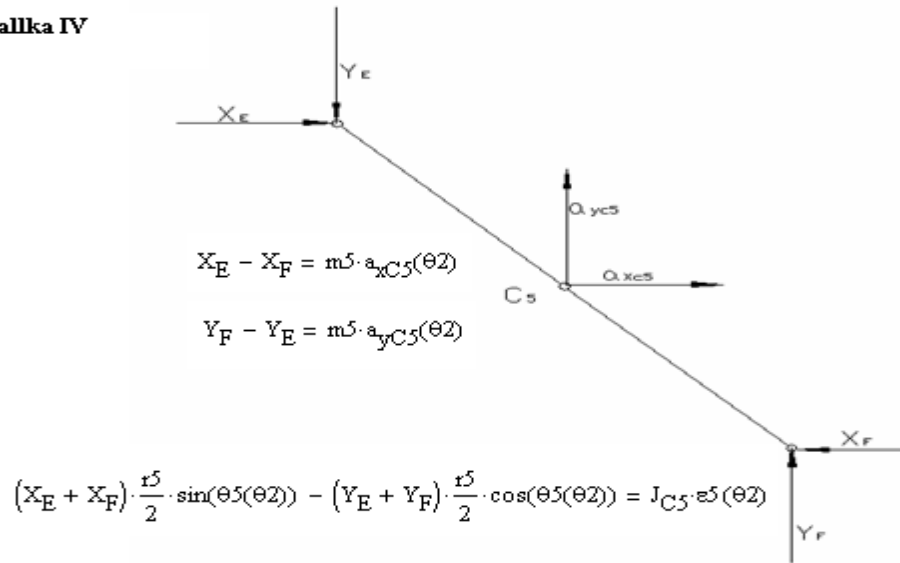
Figure 12: Linkage IV of the part *EF* in direction of *x* and *y*

Hallka V

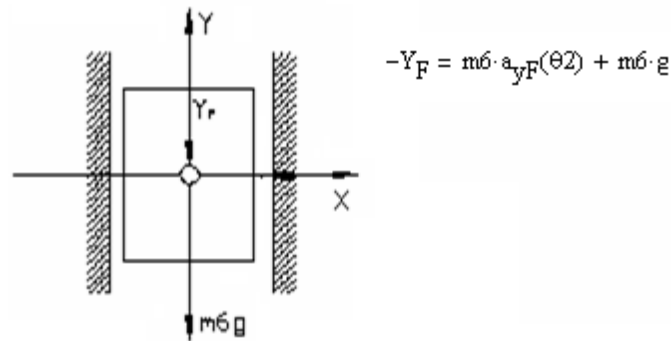$$-Y_F = m6 \cdot a_{yF}(\theta 2) + m6 \cdot g$$

Figure 13: Linkage V of the point F in direction of *x* and *y*

All unknown parameters for the linkages in both directions x and y are calculated by MathCad. From point *A* to the point *F* and the Transmission moment M_{tr}:

$$Zgj(\theta 2) := \text{Find}\left(X_A, Y_A, X_B, Y_B, X_C, Y_C, X_D, Y_D, X_E, Y_E, X_F, Y_F, M_{tr}\right)$$

$X_A(\theta 2) := Zgj(\theta 2)_0$	$Y_A(\theta 2) := Zgj(\theta 2)_1$
$X_B(\theta 2) := Zgj(\theta 2)_2$	$Y_B(\theta 2) := Zgj(\theta 2)_3$
$X_C(\theta 2) := Zgj(\theta 2)_4$	$Y_C(\theta 2) := Zgj(\theta 2)_5$
$X_D(\theta 2) := Zgj(\theta 2)_6$	$Y_D(\theta 2) := Zgj(\theta 2)_7$
$X_E(\theta 2) := Zgj(\theta 2)_8$	$Y_E(\theta 2) := Zgj(\theta 2)_9$
$X_F(\theta 2) := Zgj(\theta 2)_{10}$	$Y_F(\theta 2) := Zgj(\theta 2)_{11}$

$$M_{tr}(\theta 2) := Zgj(\theta 2)_{12}$$

In the following diagrams are given the values of all unknown parameters for the positions (0; 0.5; 1) by MathCad software. This it was really a tough work since the working process by MathCad was very slow especially to follow the procedure which is given by kinetostatic analysis.
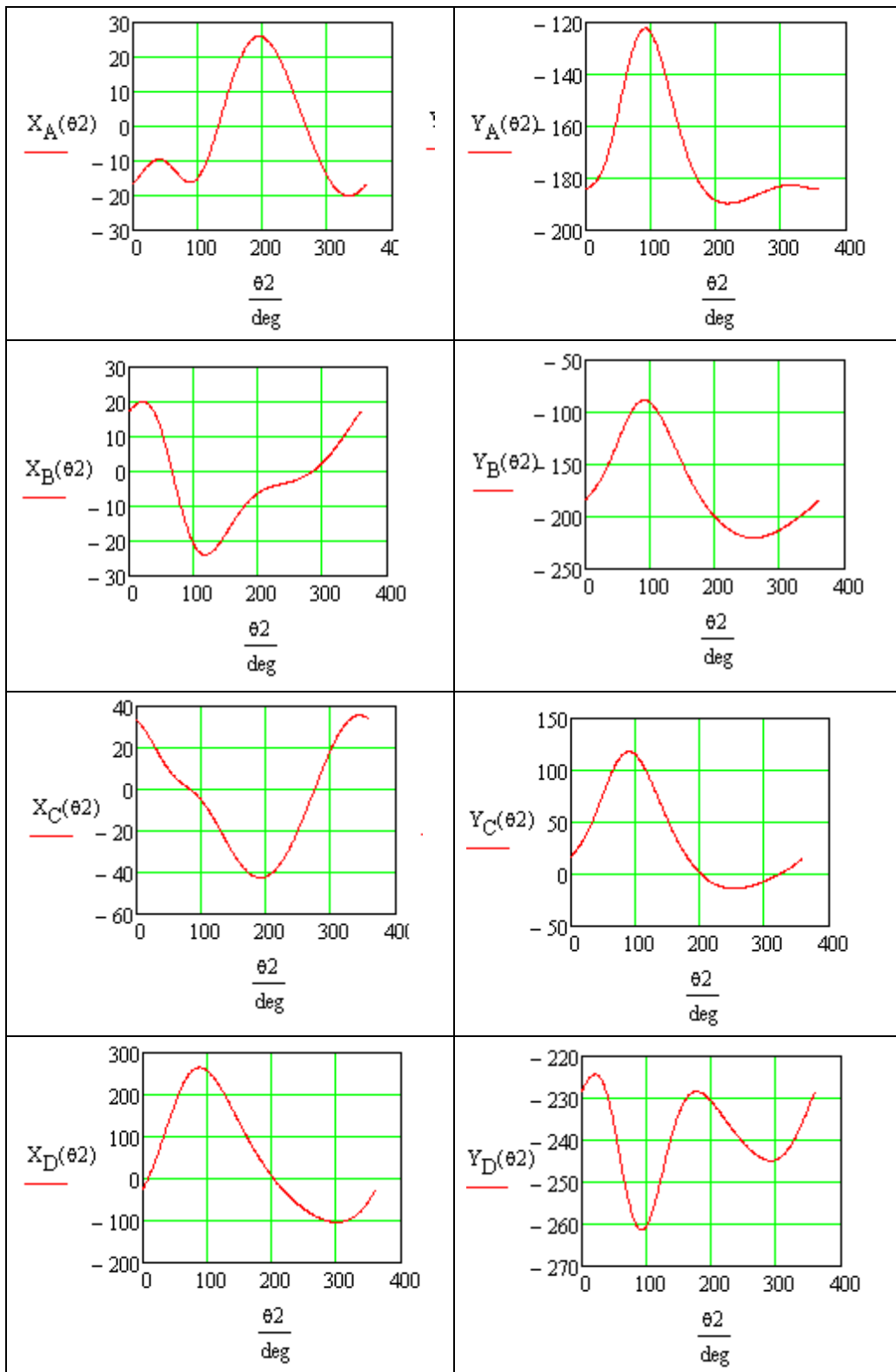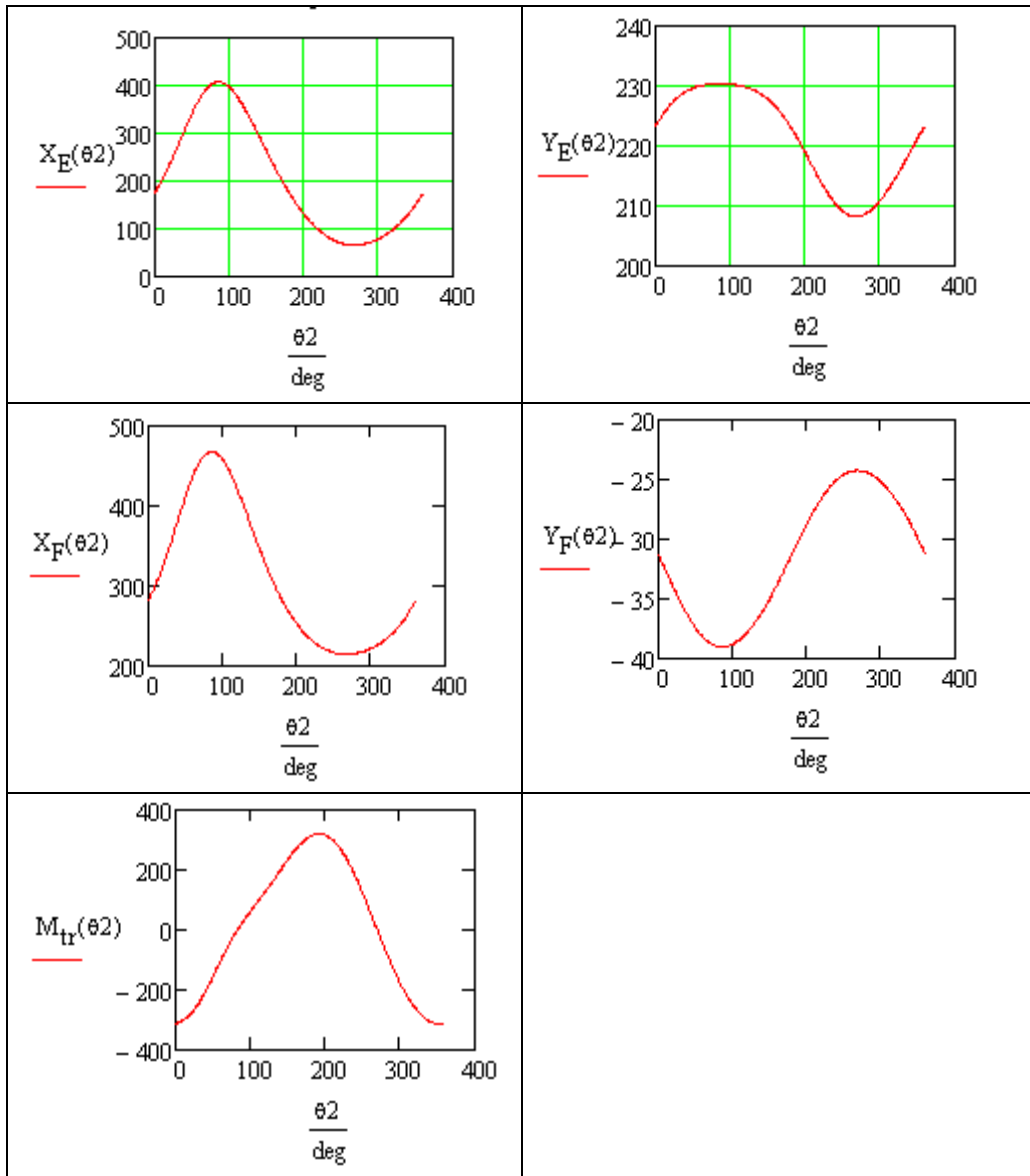
Figure 14: Diagrams of kinetostatic analysis of the given mechanism for the points A , B , C , D , E , F **and transmission moment** M_{tr}

2. Simulation for six-bar mechanism by Working Model software

In the second part of this paper is carried out the simulation for all points A , B , C , D , E , F and M_{tr} (time dependent) of the six bar linkage mechanism by Working Model, which is shown in the following.
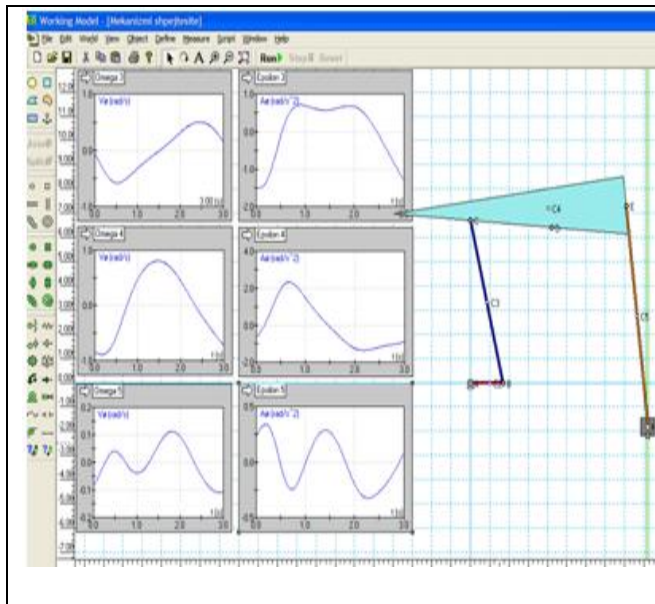
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Figure 15: Diagrams in Working Model for angular velocity and acceleration 3, 4 and 5
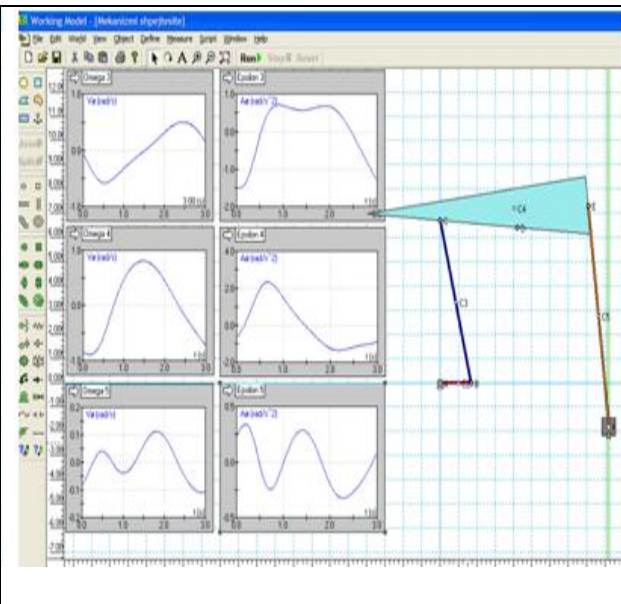
Figure 16: Diagrams in Working Model for equilibrium points A, B, C, D, E, F **and transmission moment** (M_{tr}) **in direction** x
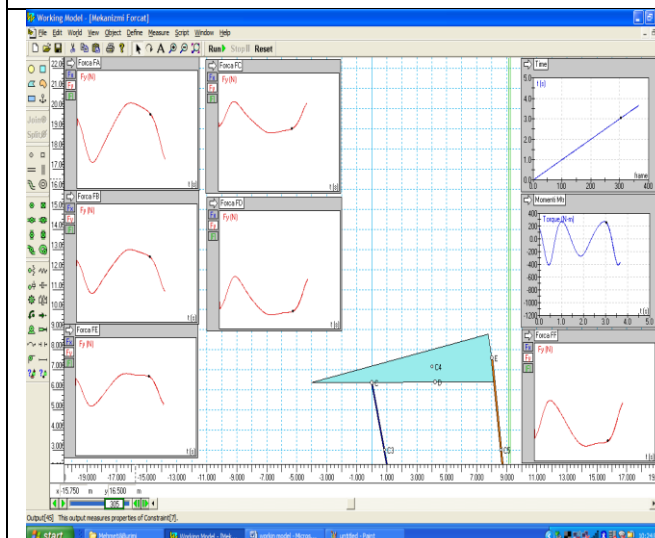
Figure 17: Diagrams in Working Model for equilibrium points A, B, C, D, E, F **and transmission moment** (M_{tr}) **in direction** y
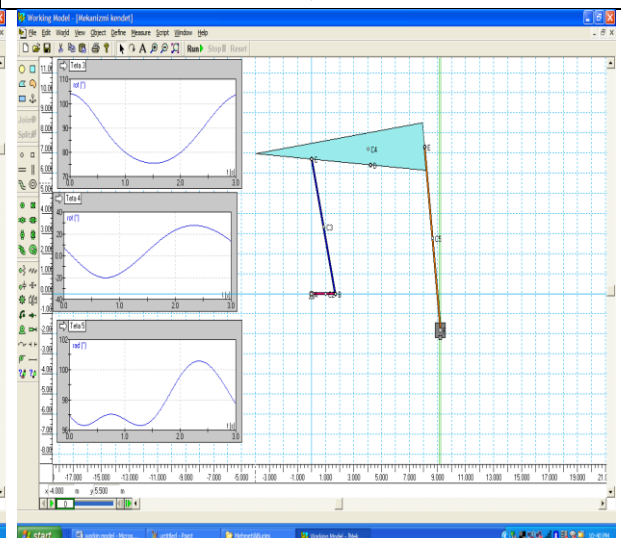
Figure 18: Diagrams for θ_3, θ_4 **and** θ_5 **(time dependent), under command RUN**

1. **Conclusions and recommendations**
   - By application of the software's MathCad or MatLab we have reached to carry out the full analysis of this paper. In this paper are made the calculations of all positions (displacement) for the whole mechanism, and also are determined the plans for velocities and accelerations for each point.

   - However, in this paper are shown the outline planes of the mechanism same as the diagrams for each linkage through MathCadand Working Model software,

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- Six-bar linkage mechanism diagrams which are derived by Working Model are almost similar to the diagrams derived by MathCad, same as the derived results,

- Through Working Model software, are derived the results of reactions from the equilibrium conditions of six bar linkage mechanism, angular velocities and accelerations for the points 3, 4 and 5, for the angles θ_3, θ_4 and θ_5 in time domain through the command Run,

- As the general conclusion; the results derived by both software, same as for their diagrams, for all points of the six bar linkage mechanism are within the reasonable boundaries. Our expectations are that similar results will be derived by application of MatLab software.

4. References

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[2] Qelaj M: (2008), Punim seminarik, Llogaritja kinematike dhe kinetostatike e mekanizmit gjashtë hallkësh me ndihmën e programit Working Model, FIM, Prishtinë,
[3] Shala A: (2008), Disenjimi i Mekanizmave me Kompjuter, Ushtrime detyrash – Fakulteti i Inxhinierisë Mekanike, Prishtinë.