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Abstract. This paper deals with the web interface Maple T.A. which is used for mathematical education at the Vienna University of Technology. Maple T.A. is based on the computer algebra system Maple. This fact offers the possibility to test and assess the students in easy but also complex mathematical matters in an uncomplicated way. This interface was first used only for the refresher course. This course takes place in the first 3 weeks of the semester and deals with the repetition of all the important themes of the school mathematics. Maple T.A. is also used for the basic and advanced mathematical courses in the first two years in the study of electrical engineering. For these courses the use of Maple T.A. includes the offer of assignments for every chapter of the lecture but also homework and tests are realized in it.

Keywords: Maple T.A., E-Learning, mathematics, exam methods, engineering education

1 Introduction

At the Vienna University of Technology Maple T.A. is used since 2008. At that time the vice rector for academic affairs asked the research group for modelling and simulation at the Institute for Analysis and Scientific Computing to organize a refresher course for the students of electrical engineering. The pegged requirement for this course included the usage of an e-learning system. Many different existing courses of other universities and various models of such courses were compared. The concept of Blended Learning fit best. Blended Learning stands for a combination of ordinary lectures and practises with e-learning.

In 2008 the first refresher course for the students of electrical engineering was offered. Since then the number of students attending the refresher courses rises every year. In 2013 around 1800 students of 6 different fields of studies were attending the refresher course.

Since 2009 Maple T.A. is also used in the basic mathematical courses for students of electrical engineering and since 2011 also in the advanced mathematical courses. These courses are located in the first, second and third semester. The courses are supported by examples on the e-learning system. In all three courses the students have to pass two of three Maple T.A. test during the semester.

Every year the number of examples in the e-learning system heightens and the complexity raise.

2 Maple TA

Maple T.A is based on the computer algebra system Maple. Maple T.A. is an interface where the creation and coordination of questions and examples is done. There are different types of question, for example multiple choice, fill in the blanks, numeric, formula and maple graded. The choice of the right question type depends on the goal of the question. There is a difference in creating examples for the students of the refresher course or for students of electrical engineering in the third semester.

For the programming of examples in Maple T.A. one can use the common commands of Maple or one can use special commands which are only working in Maple T.A.

Table 2. In the Table below the difference of the commands in Maple and Maple T.A. are shown.

	Example for programming
Maple code	if a<b then a else b end if
Embedded Maple code	$c = maple("if a<b then a else b end if")
Maple T.A. code	$c=if(gt(a,b), b, a)

Table 1 shows the variety of possibilities how to use commands either from Maple T.A. or Maple. In some cases it will be cheaper, regarding the calculation time, to use the special Maple T.A. commands. If the examples get more complex it is necessary to use embedded Maple code. For such examples it is possible to embed whole Maple procedures in a Maple T.A. variable. For example if you want to create a regular matrix you can make a small procedure which first creates a matrix, checks if it is regular and in certain cases changes it before saving it in the variable.

Another advantage of using Maple T.A. is the randomization of questions. This fact is important to guarantee that the students have the possibility to practise on the system without limit due to the number of different examples. In the program code of the examples there are randomized numbers, functions or expressions. That means that the expression in the question changes at every attempt. If someone wants to create an example dealing with integration of functions the expression to integrate will be different at every attempt. In the following figure an example is given where all the numbers and also the function *cos(x)* change.
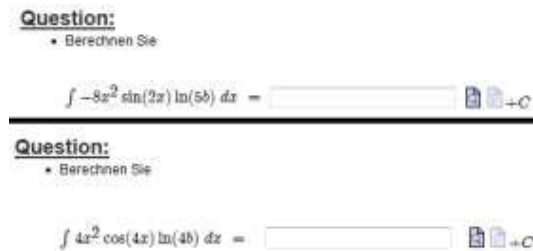

Fig. 1. In the figure a typical example is shown. The function and all the numbers except the power of x are randomized at every attempt.

Due to the raising complexity of the examples for the basic mathematical courses some libraries were developed to cope with the challenges of complex examples.

2.1 Maple Libraries

There are two important libraries for the development of the questions for the basic and advanced mathematical courses. The first library enables the developer to create complex questions in a short time. This library includes the following commands:

- Random[FromSet]
- Random[VecInts]
- Random[MatInts]
- Random[MatsIntsDef]
- Random[MatRatsEig]

All these commands support the randomisation of numbers, expressions, vectors and matrices. Random[FromSet] picks one or more expression randomly out of the given set. Random[VecInts] creates an integer vector with given dimension and limited values. Random[MatInts] creates an integer matrix with certain dimension, rank and zero counts. The last two commands specialises the characteristics of the matrix. One can choose the definition of the matrix or constrain the eigenvalues. This package focuses on problems which occur during programming examples for Linear Algebra.

The second library concentrates more on the needs of the students. It consists of several commands which enables partial grading of the examples.

- Grade[Expr], Grade[ExprDiff]
- Grade[Set]
- Grade[ParFrac]
- Grade[Vec]
- Grade[Mat]

Grade[Expr] is usually used to grade ordinary expressions by checking if the complex number or the circle constant are written in the correct way. Grade[Set] checks how many entries of the students set are right and how many are missing. If the result consists of 4 entries and the student enters only 3 and one of them is wrong then the student will get 50% of the points. Grade[ParFrac] was the reason for developing both libraries. Calculating the partial fraction expansion is very complex. One small mistake would case no points. This grading routine compares the fraction and not only the whole result. Grade[Vec] and Grade[Mat] compare the value of every entry. This library raises the acceptance of the students noticeable.

2.2 Moodle Connection

The Vienna University of Technology uses a website called TUWEL (Technische Universität Wien E-Learning) which is based on Moodle. On this website the lecturers have the possibility to create a page according to their course where all the additional materials can be stored. TUWEL also offers the administration of homework. Students can upload their homework which could be a text or a program code. In some courses the students can lop off the examples they did for homework.
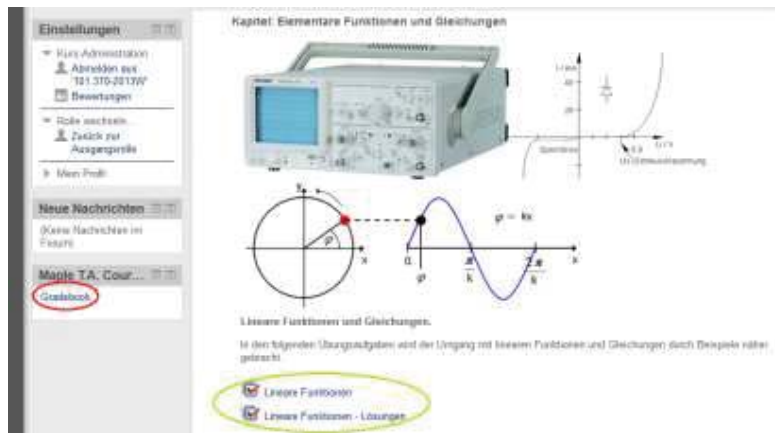

Fig. 2. This figure shows a TUWEL page of the refresher course as an example for the connection between Maple T.A. and Moodle. The links in the green circle are two assignments of Maple T.A. In the left section one can see the link to the "Gradebook" where all the results are stored.

These facilities were already used for the education of mathematics in electrical engineering. When we got to know about the possibility to connect Maple T.A. and TUWEL, the two websites we were already using, we decided to install that connection. Up to then the students had to generate a password through another website to get into Maple T.A. Additionally they got the password for the system of the university too. So they have to know different logins to use the e-learning platform. That often creates confusion.

The connection of Moodle and Maple T.A. was installed during the summer break. Since this semester the students need one single login to get to the course materials and to the e-learning system. All the examples are now embedded in the TUWEL system. Maple T.A. also offers the students to check their

grades and mistakes at any time via the "Gradebook" which is linked to TUWEL as well. The points of the tests or exams done in Maple T.A. are also stored in TUWEL. The students can check them any time. But also the lecturer can use this stored points to create the grades for the students directly in TUWEL. This connection supports lecturer as well as students.

3 Courses

3.1 Refresher Course

The refresher course was initialised in 2008. At that time this course was only offered to students of electrical engineering. Every year we tried to integrate another field of study into the course. This year we offered a separate course for 6 different fields of study and also a course for all other interested students. The course is held in two turns. One is in the last week of September, before the semester starts, the other one is in the first two weeks of October together with the semester start. This course is a voluntary offer to help students who made a break between school and university due to military or other reasons.

The refresher course consists of a lecture and an exercise part. The exercise takes place right after the lecture to strengthen the just learned mathematical facts and skills. In the exercise a tutor presents the examples and answers appearing questions. For every topic of the lecture examples in Maple T.A. exist. They are pooled in smaller lections to coordinate the students learning.
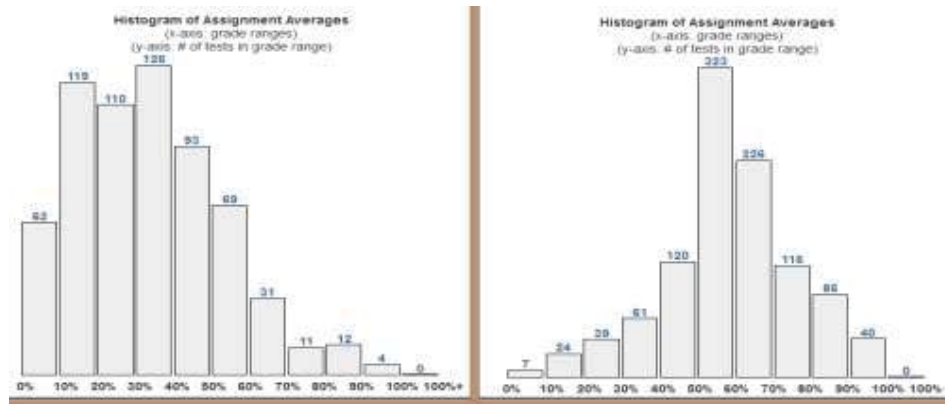

Fig. 3. In the figure some results of the refresher course are shown. The left graphic describes the results of the test at the beginning of the refresher course. The right one shows the results of the test in the end of the course.

To give the students a chance to check their knowledge an anonymous test is offered. This test is an overview regarding the mathematical level of pupils. With the help of this test the students can decide if the refresher course is necessary or not. At the beginning of the course the students have to do an entrance test to see which topics they should repeat and which topics they already know. The result of this test does not matter. There is also a test in the end of the course. The students have to pass the second test to receive a positive grade. Additionally these tests offer the possibility to observe the changes in the results of the students as shown in Figure 3.

3.2 Basic Mathematical Courses

Maple T.A. is no open software therefore we decided to use it for other lectures too. In the mathematical courses for electrical engineering in the first and second semester this e-learning system was integrated. The structure is similar to the refresher course. The courses consist of a mixture of lectures and practise. Four times a week the students have a lecture held by the professor. Once a week there is the exercise. The students have to prepare 6 to 10 examples for this exercise every week. The exercises are instructed

by a tutor. The students have to present their prepared examples with the help of the tutor. At the end of the semester the students have to prepare at least 60 percent of all the examples. Additionally to the lecture and the exercise in the class room the students have to pass two of three tests on the e-learning System Maple T.A. Every week new examples, matching to the homework of the exercises, are included in the Moodle system. These examples are available the whole semester for practicing for tests and exams. Another offer is a voluntary homework in Maple T.A. one week before the test. The results of this homework also influences the necessary points for the following test. The result of the homework counts as bonus points for the test.
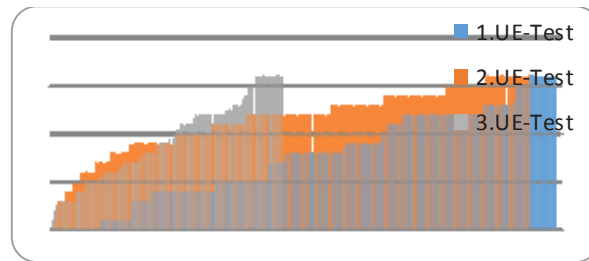

Fig. 4. This figure shows the results of the three tests spread through the second semester.

In Figure 4 the three tests of Mathematics 2 are shown. The blue one was the first test of the semester. Some of the students have not attended the refresher course and had to learn how to handle Maple T.A. The red one represents the second test. An increasing of the total points of the second test can be seen. The last test of the semester, the green one, was only made by half of the students due to the constraint to pass two of three tests. To receive a positive grade for the exercise the students have to pass 2 tests, prepare 60 percent of the examples and present them in a good way. For the lecture the students get a separate mark after passing a written and oral exam. The written part can be made in an ordinary way on paper or in Maple T.A. This exam verifies the calculation skills. With a positive written exam the students have an oral exam where the professor checks the theoretical background of the students.

3.3 Advanced Mathematical Courses

In the third semester Mathematics 3 takes place. It also consists of a lecture and an exercise. In this course the exercise is a combination of the exercise of the refresher course and the basic mathematical courses. It is only every second week. The students get the examples one week before and can present them voluntary. Otherwise the tutor presents the examples and answers occurring questions.

In this course there are also three tests during the semester. As in the basic mathematical courses the students have to pass two of them. One week before the test takes place a homework similar to the test is offered. This homework should activate the students who would have started learning one day before the test. The homework arrangement is an improvement which will be evaluated after this year. We hope that there will be a positive impact compared to the years before.

The students only get one mark for this course. It is necessary to pass two tests but the presentation of examples during the exercise and the points from the homework bring bonus points. There is only an oral exam. Nevertheless the professor gets all the information about the provided performances of every student.

3.4 Examples

In the following paragraph some examples of the different levels of the courses will be explained. The first example is one of the refresher courses. The example in Figure 5 shows a sine or cosine with a certain amplitude, frequency and shift up or down. To provide confusions due to the coordinate ystem all the coefficient are integers. The recognition of curves is very important in all technical studies.
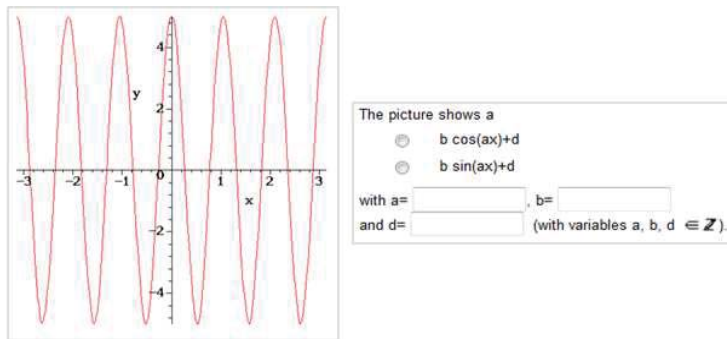
The picture shows a
- ○ b cos(ax)+d
- ○ b sin(ax)+d

with a= [   ] , b= [   ]

and d= [   ]   (with variables a, b, d $\in \mathbb{Z}$).

Fig. 5. The example above is taken out of the refresher course. It shows a cosine and the students have to determine the certain coefficients.

The second example deals with differential equations. The equation at the beginning of the example describes a certain jacobi field. The students have to match the equation to the right jacobi field.
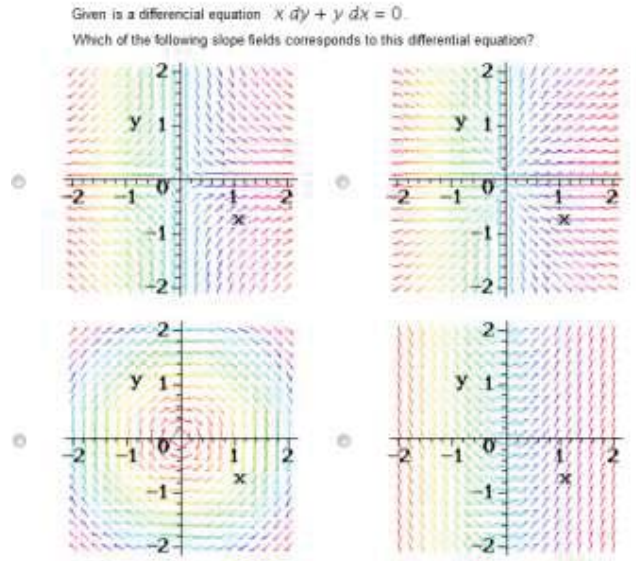

Given is a differencial equation $x\,dy + y\,dx = 0$.

Which of the following slope fields corresponds to this differential equation?

Fig. 6. The figure shows an example of the basic mathematical course in the first semester. The students have to match the differential equation with the according jacobi field.

The last example in Figure 7 is one of the questions of the Linear Algebra. The students have to pick a vector out of the kernel of the matrix. That means that the chosen vector multiplied by the matrix results zero. In this special figure one can see the effect using the specially developed library. The chosen vector was graded with a quarter of the points because only the multiplication with the first row gives zero due to the zero entry in the third column of the matrix.

Fig. 7. This example deals with Linear Algebra and is used in the second semester.

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