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Leka, Hizer and Kabashi, Faton, "Incidence matrix and some of its applications in graph theory" (2021). *UBT International Conference*. 399.
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Incidence matrix and some of its applications in graph theory

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Abstract

In this paper we will focus mainly on some basic concepts and definitions regarding incidence matrices and some examples of their application in graph theory. To give their clearest definition of the incidence matrix, we will first give the meaning of the incidence structure, then through it to define the incidence matrix.

The structure of incidence is called the ordered triplet $S = (P, B, I)$, where $P \cap B = \phi, I \subseteq P \times B$ and P, B while, are two non-empty sets and I a relation in between them, such that $I \subset P \times B$. We call the elements of P community dots and we will mark them in lower case letters of, and we will call B the elements of the community blocks or lines and we will mark them in uppercase letters. Like any double bond, between two finite sets the incidence I bond of a finite structure $S = (P, B, I)$ has the bond matrix, which we call the incidence matrix. The incidence matrix A represents a reflection of $P \times B \to \{0,1\}$, that is $(p, X) \longrightarrow 1$, if $p\, I\, X$ and $(p, X) \longrightarrow 0$, if $p \nmid X$ and is denoted $A = (a_{ij})_{vxb}$. If G is a graph with n vertices, m edges and without self-loops. The incidence matrix A of G is an $n \times m$ matrix $A = (a_{ij})$ whose n rows correspond to the n vertices and the m columns correspond to m edges such that:

$$A = (a_{ij}) = \begin{cases} 1, & if\ j^{th}\ edge\ m_j\ is\ incident\ on\ the\ i^{th}\ vertex; \\ 0, & otherwise. \end{cases}$$

Incidence matrices have a great application in many fields of science such as: telecommunications, coding theory, graph theory, etc.

Keywords

Matrices, incidence, graph, rank, cycle matrix, cut-set matrix

1. Introduction

Let be P, B two idle communities and I a relation between them, such that $I \subset P \times B$.

Definition 1.[8] The ordered trinity $S = (P, B, I)$, where

$$\mathrm{P} \cap \mathrm{B} = \phi, I \subseteq P \times B$$

is called the incidence structure.

$S = (P, B, I)$ is the structure of incidence, if P, B are distinct sets (disjuncts) and I is a double (binary) relation of the P community to the B community, called the incidence relation. We call the elements of a P community dots and we will mark them in lower case letters of the alphabet, while we will call the B elements of the community blocks (or straight lines) and we will mark them in capital letters of the alphabet. As in any double bond, the fact $(p, X) \in I$ can be denoted by $p\ I\ X$ we will read: point p is incident with block X or block X has incident point p.

Like any double bond between two finite sets, the incidence bond I of a finite structure $S = (P, B, I)$ has the bond matrix, which in this case we call the incidence matrix.

2. Incident matrices

Definition 2. [6] Let S be the incidence structure with v points and b blocks, where $P = \{p_1, p_2, \ldots, p_v\}$ and $B = \{X_1, X_2, \ldots, X_b\}$.

Matrix

$$A = (a_{ij}) = \begin{cases} 1, if\ p_i\ I\ X_j \\ 0, if\ p_i \nmid X_j \end{cases}$$

is called the incidence matrix for the structure S.

The incidence matrix A represents a reflection from $P \times B \to \{0,1\}$, i.e. $(p, X) \to 1$, if $p\ I\ X$ and $(p, X) \to 0$, if $p \nmid X$ (p is not an incident with X), i.e. $A = (a_{ij})_{vxb}$.

This matrix is obtained as follows:

In a rectangular table on the left, placed vertically the starting set P with its v points and above horizontally placed the arrival set X with its k blocks. The empty rows and columns of the table are filled with 1 or 0 respectively when $(P_i, b_j) \in I$ is marked 1 and when $(P_i, b_j) \in I$ not element I is marked 0.

The matrix with $v \times k$, obtained in this way, is the incidence matrix. We can study the nature of a finite structure through its incidence matrix. It is clear that the sum of 1^s in its i^{th} row is the power of the point P_i, whereas the sum of 1^s in its j^{th} column is the power of block b_j.

Example 1. Let be $S = (P, B, \in)$ a finite structure with:

$P = \{1,2,3,4,5,6,7\}$ and $B = \{b_1, b_2, b_3, b_4, b_5, b_6, b_7\}$, where

$b_1 = \{1,2,4\}$, $b_2 = \{2,3,5\}$, $b_3 = \{3,4,6\}$, $b_4 = \{4,5,7\}$, $b_5 = \{1,5,6\}$, $b_6 = \{2,6,7\}$, $b_7 = \{1,3,7\}$.

The incidence matrix for this structure will be:

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

In it, each row has 3 units, so all points have the same power $r = 3$, which indicates that the structure is regular. But even each column has 3 units, so it is also uniform. Hence it is a tactical configuration.

3. Graph and rank of incidence matrix

Let G be a graph with n vertices, m edges and without self-loops.

Definition 3. [5] The incidence matrix A of G is an $n \times m$ matrix $A = [a_{ij}]$, whose n rows correspond to the n vertices and the m columns correspond to m edges such that:

$$A = (a_{ij}) = \begin{cases} 1, & if\ p_i\ I\ X_j \\ 0, & otherwise \end{cases}$$

It is also called vertex-edge incidence matrix and is denoted by $A(G)$.

Example 2. Consider the graphs given in figure 1.

The incidence matrix of G_1 is:

$$A(G_1) = M_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{matrix} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \end{matrix} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

The incidence matrix of G_2 is:

$$A(G_2) = M_2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{matrix} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{matrix} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

The incidence matrix of G_3 is:

$$A(G_3) = M_3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

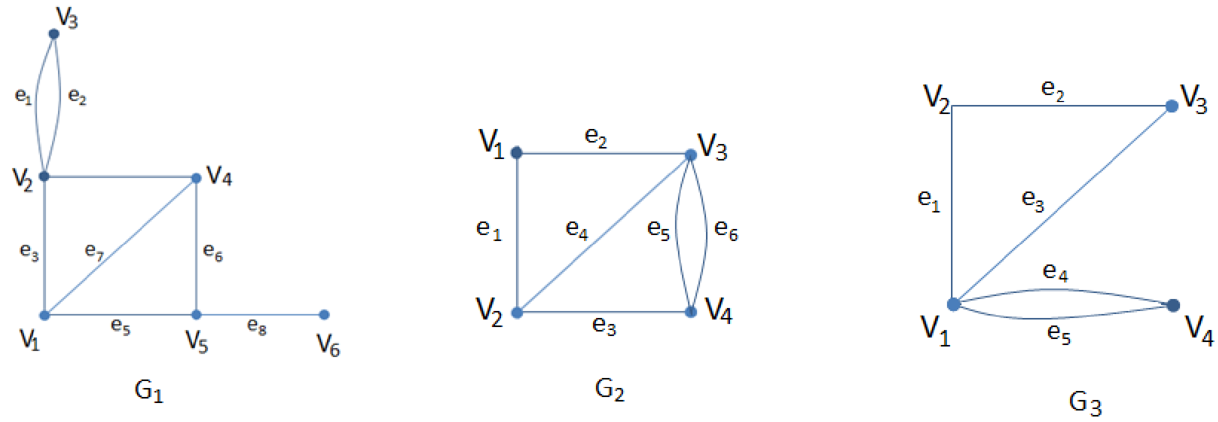


Fig.1

Let G be a graph and let $A(G)$ be its incidence matrix. Now each row in $A(G)$ is a vector over $GF(2)$ in the vector space of graph G. Let the row vectors be denoted by $A_1, A_2, \ldots, A_n$. Then,

$$A(G) = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_n \end{bmatrix}$$

since there are exactly two 1^s in every column of A, the sum of all these vectors is 0 (this being a modulo 2 sum of the corresponding entries). Thus vectors $A_1, A_2, \ldots, A_n$ are linearly dependent. Therefore, rank $A < n$. Hence, rank $A < n - 1$. [2]

Let H be a subgraph of a graph G, and let $A(H)$ and $A(G)$ be the incidence matrices of H and G respectively. Clearly, $A(H)$ is a submatrix of $A(G)$, possibly with rows or columns permuted. We observe that there is a one-one correspondence between each $n \times k$ submatrix of $A(G)$ and a subgraph of G with k edges, k being a positive integer, $k < m$ and n being the number of vertices in G.

The following is a property of the submatrices of $A(G)$.

Theorem 1. [4] Let $A(G)$ be the incidence matrix of a connected graph G with n vertices. $A_n = (n-1) \times (n-1)$ submatrix of $A(G)$ is non-singular if and only if the $n - 1$ edges corresponding to the $n - 1$ columns of this matrix constitutes a spanning tree in G.

The following is another form of incidence matrix.

Definition 4.[2] The matrix $F = [\,f_{ij}]$ of the graph $G = (V, E)$ with $V = \{v_1, v_2, \ldots, v_n\}$ and $E = \{e_1, e_2, \ldots, e_m\}$, is the $n \times m$ matrix associated with a chosen orientation of the edges of G in which for each $e = (v_i, v_j)$, one of v_i or v_j is taken as positive end and the other as negative end, and is defined by:

$$f_{ij} = \begin{cases} 1, & if\ v_i\ is\ positive\ end\ of\ e_j, \\ -1, & if\ v_i\ is\ the\ negativ\ end\ of\ e_j, \\ 0, & if\ v_i\ is\ not\ incident\ with\ e_j. \end{cases}$$

This matrix F can also be obtained from the incidence matrix A by changing either of the two 1^s to -1 in each column.
The above arguments amount to arbitrarily orienting the edges of G, and F is then the incidence matrix of the oriented graph.
The matrix F is then the modified definition of the incidence matrix A.

Example 3.[3] Consider the graph G shown in *figure* 3, with $V = \{v_1, v_2, v_3, v_4, v_5\}$ and $E = \{e_1, e_2, e_3, e_4, e_5\}$.

The incidence matrix is given by:

$$A(G) = \begin{matrix} & e_1 & e_2 & e_3 & e_4 & e_5 \\ v_1 & 1 & 0 & 0 & 1 & 0 \\ v_2 & 1 & 1 & 0 & 0 & 0 \\ v_3 & 0 & 1 & 1 & 0 & 0 \\ v_4 & 0 & 0 & 1 & 1 & 1 \\ v_5 & 0 & 0 & 0 & 0 & 1 \end{matrix}$$

Therefore,

$$F = (f_{ij}) = \begin{matrix} & e_1 & e_2 & e_3 & e_4 & e_5 \\ v_1 & 1 & 0 & 0 & 1 & 0 \\ v_2 & -1 & 1 & 0 & 0 & 0 \\ v_3 & 0 & -1 & 1 & 0 & 0 \\ v_4 & 0 & 0 & -1 & -1 & 1 \\ v_5 & 0 & 0 & 0 & 0 & -1 \end{matrix}$$

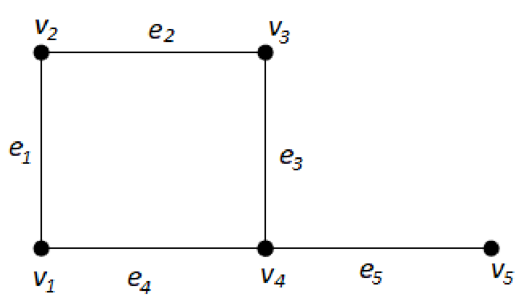


Fig.3

Let the graph G have medges and let q be the number of different cycles in G.

Definition 5. [4] The cycle matrix $C = [c_{ij}]_{q\times m}$ of G is a $(0,1)-$ matrix of order $q \times m$, with $c_{ij} = 1$, if the i^{th} cycle includes j^{th} edge and $c_{ij} = 0$, otherwise. The cycle matrix C of a graph G is denoted by $C(G)$.

Example 4. Consider the graph G given in figure 4.

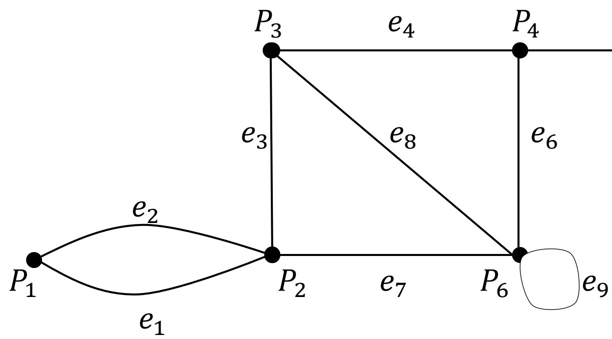


Fig.4

The graph G has five different cycles: $X_1 = \{e_1, e_2\}, X_2 = \{e_3, e_4, e_6, e_7\}, X_3 = \{e_3, e_7, e_8\}$, $X_4 = \{e_4, e_6, e_8\}$ and $X_5 = \{e_9\}$. The cycle matrix is:

$$C(G) = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array} \begin{array}{c} \begin{array}{ccccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Example 5. Consider the graph G given in figure 5.

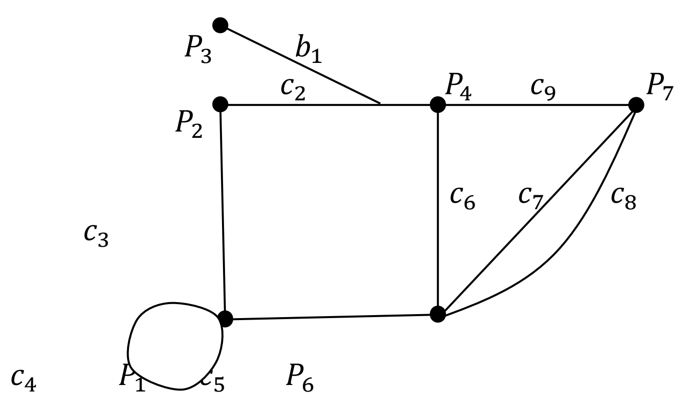


Fig. 5.

The graph G of *figure 5*., has seven different cycles namely, $Y_1 = \{c_2, c, c_5, c_6\}$, $Y_2 = \{c_6, c_7, c_9\}$, $Y_3 = \{c, c_8, c_9\}$, $Y_4 = \{c_7, c_8\}$, $Y_5 = \{c_2, c_3, c_5, c_7, c_9\}$, $Y_6 = \{c_2, c_3, c_5, c_8, c_9\}$ and $Y_7 = \{c_4\}$. The cycle matrix is given by:

$$C(G) = \begin{matrix} & c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 \\ y_1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ y_2 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ y_3 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ y_4 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ y_5 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ y_6 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ y_7 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

If the graph G is separable (or disconnected) and consists of two blocks (or components) $H1$ and $H2$, then the cycle matrix $C(G)$ can be written in a block-diagonal form as:

$$C(G) = \begin{bmatrix} C(H_1) & 0 \\ 0 & C(H_2) \end{bmatrix}$$

where $C(H1)$ and $C(H2)$ are the cycle matrices of $H1$ and $H2$. This follows from the fact that cycles in $H1$ have no edges belonging to $H2$ and vice versa. [3]

Let $C_1, C, \ldots, C_k$ be the cycle matrices of $G1, G2, \ldots, Gk$.

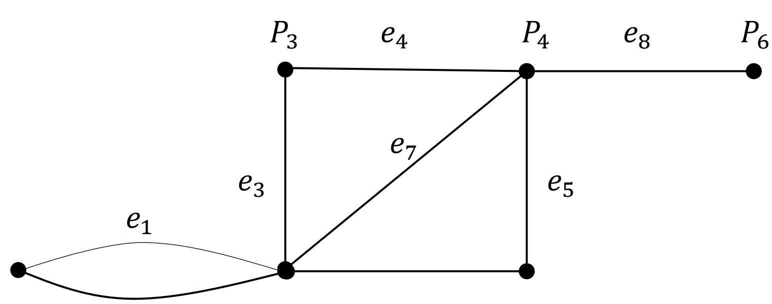
Then,

$$C(G) = \begin{bmatrix} C_1(G_1) & 0 & 0 & \cdots & 0 \\ 0 & C_2(G_2) & 0 & \cdots & 0 \\ 0 & 0 & C_3(G_3) & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & C_k(G_k) \end{bmatrix}$$

We know rank $C_i = m_i - n_i + 1$, for $1 < i < k$.

Therefore, $rank\ C = rank\ C_1 + rank\ C_2 + \cdots + rank\ C_k = (m_1 - n_1 + 1) + \cdots + (m_k - n_k + 1) = (m_1 + m_2 + \cdots + m_k) - (n_1 + n_2 + \cdots + n_k) + k = m - n + k$. [3]

Theorem 2. [1] If G is a graph without self-loops, with incidence matrix A and cycle matrix C whose columns are arranged using the same order of edges, then every row of C is orthogonal to every row of A, that is $AC^T = CA^T \equiv 0$ (mod2), where A^T and C^T are the transposes of A and C respectively.
We illustrate the above theorem with the following example (Fig. 6). [3]



P_1 e_2 P_2 e_6 P_5

Fig.6

Cleary,

$$A(G)=\begin{bmatrix}1&1&0&0&0&0&0&0\\1&1&1&0&0&1&1&0\\0&0&1&1&0&0&0&0\\0&0&0&1&1&0&1&1\\0&0&0&0&1&1&0&0\\0&0&0&0&0&0&0&1\end{bmatrix} \text{ and } C(G)=\begin{bmatrix}1&1&0&0&0&0&0&0\\0&0&1&1&0&0&1&0\\0&0&0&0&1&1&1&0\\0&0&1&1&1&1&0&1\end{bmatrix}$$

There,

$$A\cdot C^T=\begin{bmatrix}1&1&0&0&0&0&0&0\\1&1&1&0&0&1&1&0\\0&0&1&1&0&0&0&0\\0&0&0&1&1&0&1&1\\0&0&0&0&1&1&0&0\\0&0&0&0&0&0&0&1\end{bmatrix}\cdot\begin{bmatrix}1&1&0&0&0&0&0&0\\0&0&1&1&0&0&1&0\\0&0&0&0&1&1&1&0\\0&0&1&1&1&1&0&0\end{bmatrix}^T=$$

$$=\begin{bmatrix}1&1&0&0&0&0&0&0\\1&1&1&0&0&1&1&0\\0&0&1&1&0&0&0&0\\0&0&0&1&1&0&1&1\\0&0&0&0&1&1&0&0\\0&0&0&0&0&0&0&1\end{bmatrix}\cdot\begin{bmatrix}1&0&0&0\\1&0&0&0\\0&1&0&1\\0&1&0&1\\0&0&1&1\\0&0&1&1\\0&1&1&0\\0&0&0&0\end{bmatrix}=\begin{bmatrix}2&0&0&0\\2&2&2&2\\0&2&0&2\\0&2&2&2\\0&0&2&2\\0&0&0&0\end{bmatrix}=$$

$$=2\cdot\begin{bmatrix}1&0&0&0\\1&1&1&1\\0&1&0&1\\0&1&1&1\\0&0&1&1\\0&0&0&0\end{bmatrix}\equiv 0(mod2) \qquad \dots \qquad (1)$$

Also,

$$C\cdot A^T=\begin{bmatrix}1&1&0&0&0&0&0&0\\0&0&1&1&0&0&1&0\\0&0&0&0&1&1&1&0\\0&0&1&1&1&1&0&0\end{bmatrix}\cdot\begin{bmatrix}1&1&0&0&0&0&0&0\\1&1&1&0&0&1&1&0\\0&0&1&1&0&0&0&0\\0&0&0&1&1&0&1&1\\0&0&0&0&1&1&0&0\\0&0&0&0&0&0&0&1\end{bmatrix}^T=$$

$$= \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 2 & 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 2 & 2 & 0 \end{bmatrix} = 2 \cdot \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix} \equiv 0 (mod 2) \qquad \ldots\ (2)$$

From (1) and (2), drives: $A \cdot C^T = C \cdot A^T \equiv 0\ (mod\ 2)$.

Definition 4. [1] Let G be a graph with m edges and p cutsets. The cut-set matrix $Q = [q_{ij}]_{p\times m}$ of G is a $(0,1)$ −matrix with

$$q_{ij} = \begin{cases} 1, & if\ i^{th}\ cut-set\ contrains\ j^{th}\ edge, \\ 0, & othewise. \end{cases}$$

Example 6. Consider the graphs shown in figure 8.

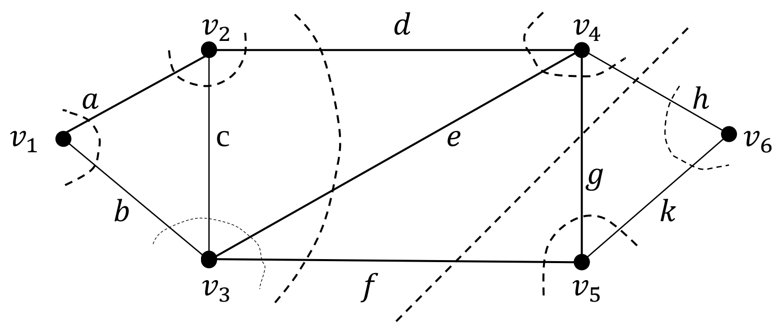


Fig. 8

In the graph $G, E = \{a, b, c, d, e, f, g, h, k\}$.
The cut-sets are: $q_1 = \{a, b\}, q_2 = \{a, c, d\}, q_3 = \{d, e, f\}, q_4 = \{f, g, h\}, q_5 = \{f, g, k\}$, $q_6 = \{b, c, e, f\}, q_7 = \{d, e, g, h\}\ and\ q_8 = \{h, k\}$.

Thus the cut-set matrices are given by:

$$Q(G) = \begin{matrix} & a & b & c & d & e & f & g & h & k \\ q_1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ q_2 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ q_3 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ q_4 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ q_5 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ q_6 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ q_7 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ q_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{matrix}$$

Conclusion

1. The incidence matrix represents the relation of an incidence I of a finite structure $S = (P, B, I)$.
2. The incidence matrix graph is easily constructed, based only on its definition.
3. Incidence matrices are applied in many scientific fields such as: telecommunications, graph theory, coding theory, computer science, statistics, etc.
4. The matrix A has been defined over a field, Galois field modulo 2 or GF(2), that is, the set {0, 1} with operation addition modulo 2.

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