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Mathematics Unplugged and Digitized: Bridging Traditional and Tech-Driven Approaches

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Abstract

The paper starts with analyzing a real world example that was given as a project in algebra class to apply the piecewise functions in the Giant's Devil's Flower Mantis (two spiked grasping forelegs), and reviewing literature on the combining technology in education. Digital technology graphs the parabola within the insect and adjusts the critical points based on the error and trial (for parabola). Also, the technology accurately graphs the linear equations representing the antenna of the insect by connecting two points. Technological tools (desmos - graphing calculator) determine the piecewise functions that describe the upper body of the insect that is depicted in Igor Siwanowicz photography. In the same figure is applied analytical work to verify results of the technology, in addition, the analytical work confirms the same result with the tenth point decimal accuracy. Combining the work of technology and critical thinking proves that combining teaching mathematics with and without technology increases the impact of the instructions and elevates the learning outcome of high school students. Moreover, the paper applies inductive analytical research by analyzing the effect of combining teaching with and without technology through reviewing literature. Consequently, the literature confirms the students prefer classes with moderate IT to other four different options. Above all, the paper brings the evidence that combining teaching with and without technology advances instructions to a level where teachers and students could create conjectures, respectively theorems.

Key Words: Technology, Critical Thinking, Rationally, Combining, Instruction, Learning, Students

1. Introduction

The paper aims to delineate a rational approach to instruction by combining teaching with and without technology, utilizing a real-world example (an algebra class project) as the initial scenario. This is followed by additional examples from the literature that illustrate this methodology in the broader context of teaching mathematics. Crucially, the research is grounded in an analytical induction design.

The strategic amalgamation of technology and traditional teaching methods in mathematics markedly enhances students' learning outcomes. The integration of technology into mathematics education involves the invaluable utilization of tools during instructional processes. Technology empowers educators and learners to visualize geometric figures and graphs on an exact scale with intricate detail. This heightened precision, as demonstrated by Hanson (2019), has catalyzed significant mathematical advancements. Furthermore, technology facilitates swift result verification and validation of mathematical work.

Conversely, math educators are encouraged to employ critical thinking when elucidating mathematical concepts or solving mathematical problems through non-technological means. The math teacher should resort to technological tools when explicating intricate mathematical concepts or resolving complex problems. However, in situations where technology is unnecessary, traditional teaching methods are still advantageous.

The paper outlines the efficacy of combining mathematical instruction with and without technology through the analysis of various scenarios. The notion of using technology as a pedagogical tool within the mathematics classroom holds significant importance in the context of this exploratory study (Nkhwalume, 2013). The provided examples encompass situations wherein technology clarifies natural phenomena, enabling the application of mathematical strategies commonly employed in high school math, particularly in algebra.

One example illustrates the utilization of piecewise functions (quadratic and linear functions) in the context of an insect's characteristics. Additionally, this example highlights how technology serves as a tool for validating and verifying results. Another case involves a math teacher formulating a conjecture by observing examples during instructional sessions aimed at solving mathematical problems. In this instance, technology expedites result generation and aids in arriving at a decision to assert the conjecture. Subsequently, it's revealed that the conjecture evolves into a proven theorem.

To prevent inaccuracies, the utilization of technology becomes essential when encountering obstacles. The amalgamation of modern and traditional teaching methods serves to mitigate shortcomings and elevate instruction to a novel echelon. Numerous challenges emerge in geometry and geometrical interpretation, often stemming from the misrepresentation of figures due to inaccurately scaled drawings. This practice can lead to a distortion where geometric shapes are erroneously grouped under unrelated categories. The mental imagery of the object under scrutiny, however, significantly influences one's perception while attempting to materialize the sought-after object (Brown, 2015).

In contemporary times, mathematicians employ technology even for substantiating intricate mathematical conjectures, involving extensive calculations such as lemmas and theorems. Technology's expansive role in mathematics yields enhanced comprehension and improved outcomes. Nonetheless, technology brings along its drawbacks, fostering dependence and impeding critical thinking. The optimal approach to circumvent mathematical inaccuracies and foster sharper critical thinking involves a harmonious blend of both techniques – traditional teaching without technology and the strategic use of technology when required.

2. Method

The paper will center on the astute integration of technology within high school mathematical instruction, particularly in instances where technology expedites instructions without feasible substitutes, alongside a teaching methodology devoid of technology that nurtures critical thinking. The instruction of select mathematical concepts sans technology yields a more pronounced impact on the cultivation of students' critical thinking abilities.

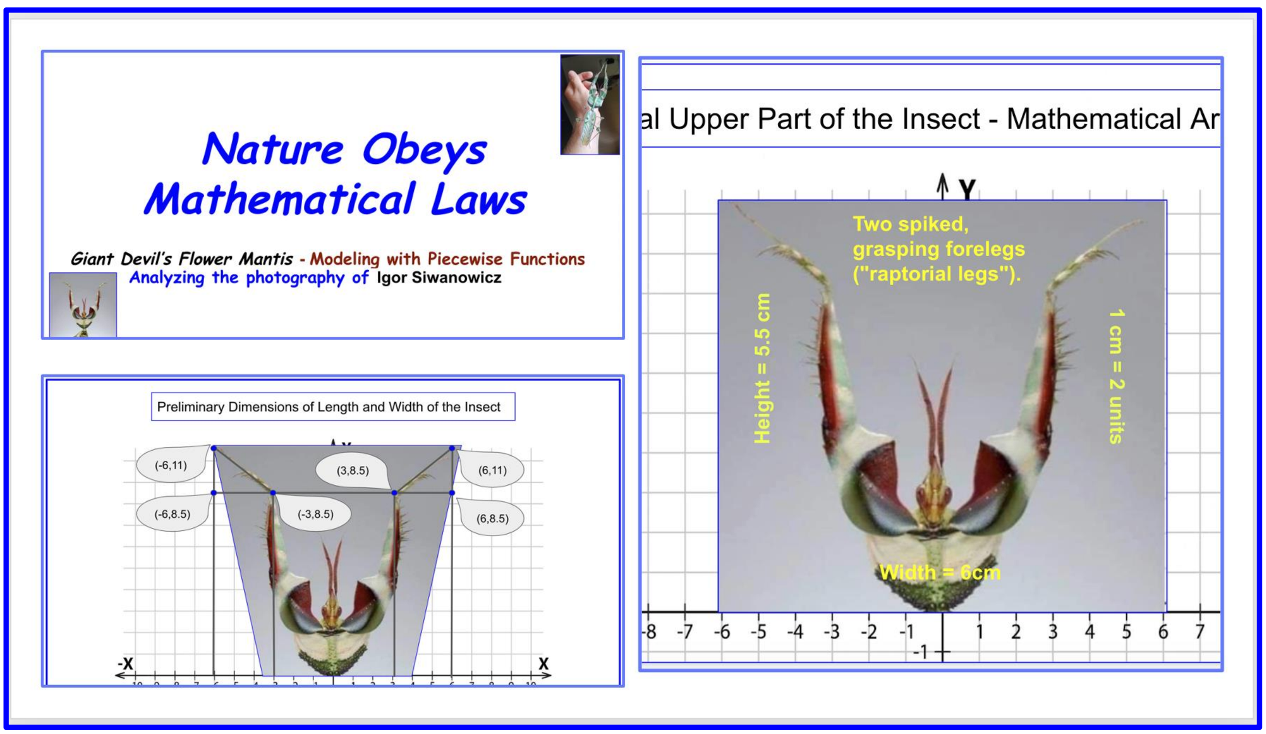

***Figure 1**. Analyzing the photography of Igor Siwanowics by applying the piecewise well defined functions in the Giant Devil's Flower Mantis.*

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For instance, numerous examples in standardized exams can be resolved without calculators more expeditiously than with their use. Conversely, scenarios exist—such as deciphering figures from real-world contexts or intricate geometrical shapes—where technology serves as an invaluable conduit for lucidly explicating mathematical concepts. Devoid of technology, comprehending the content swiftly and thoroughly becomes exceedingly challenging.

Thus, a rational approach to teaching necessitates the incorporation of modern educational technology. This transcends mere utilization of contemporary tools in the classroom; it is paramount to assist students in conquering pivotal and intricate aspects of knowledge comprehension (Wang et al., 2010). This approach is evident even in standardized tests like the SAT/ACT, where the mathematics section is divided into two parts: one allowing calculators and the other disallowing them. The SAT exam comprises four sections, with the third being a calculator-free mathematics exam and the fourth involving calculator use. Logically, these standardized math exam sections correspond with the overarching structure of the math curriculum. However, the question content necessitates heightened critical thinking from slightly different perspectives.

In essence, the fusion of instructional methods involving both technology and non-technology approaches is inevitable, fostering a comprehensive educational experience.

2.1. Review on the single Project

Based on general information, the Giant Devil's Flower Mantis typically attains a size of about 5 inches. More specifically, females reach a length of around 13 centimeters (approximately 5 inches), while males grow to about 10 centimeters (approximately 4 inches) (All species Wiki, n. d.). These dimensions are conveniently illustrated in Figure 1.

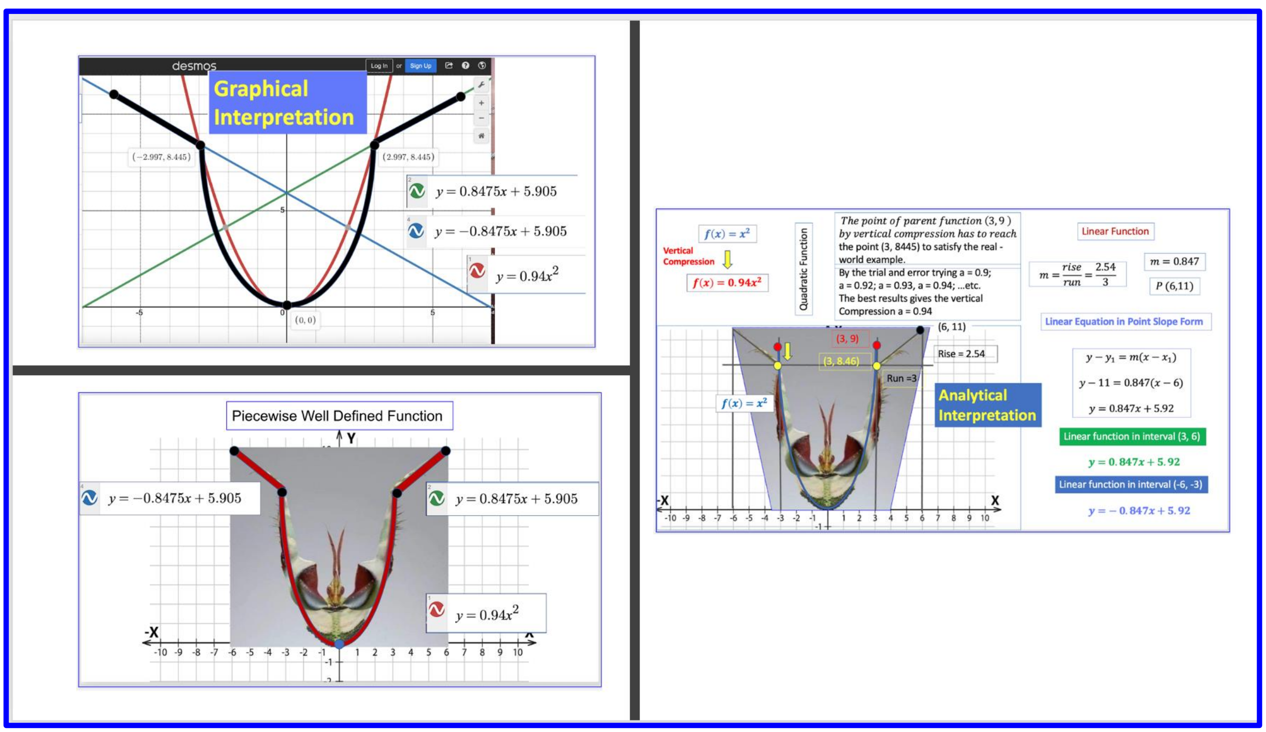

__Figure 2__. Application of technology on interpreting graphically and application of analytical work based on critical thinking (the work without technology).

Numerous real-world problems can be effectively represented through mathematical models. However, it's important to note that a model may not capture the real-world situation with absolute precision (Envision Algebra, 2). In the present example, each square in both the horizontal and vertical directions corresponds to a length of 0.5 centimeters. When the insect raises its two spiked grasping forelegs, as depicted in Igor's photography, its body exhibits an exquisite symmetry with respect to the y-axis, presenting a visually pleasing form. Thus, I positioned the midpoint of the upper body part at the coordinate system's origin.

Upon visual inspection of the figure (without employing technology), it becomes apparent that the shape can be likened to a piecewise-defined function, comprising parabolic and linear components.

Problem-solving in mathematics hinges on the proficient application of analytical methods devoid of technology. While solving mathematical problems analytically might be manageable, grappling with intricate geometrical figures, including real-world shapes or graphs, can be perplexing. Technology is a judiciously applicable resource across various high school subjects. Whether it's English language arts, mathematics, sciences, social studies, history, art, or music, integrating 21st-century competencies and skills such as critical thinking, complex problem-solving, collaboration, and multimedia communication is imperative in all subject areas (National Education Technology, 2010).

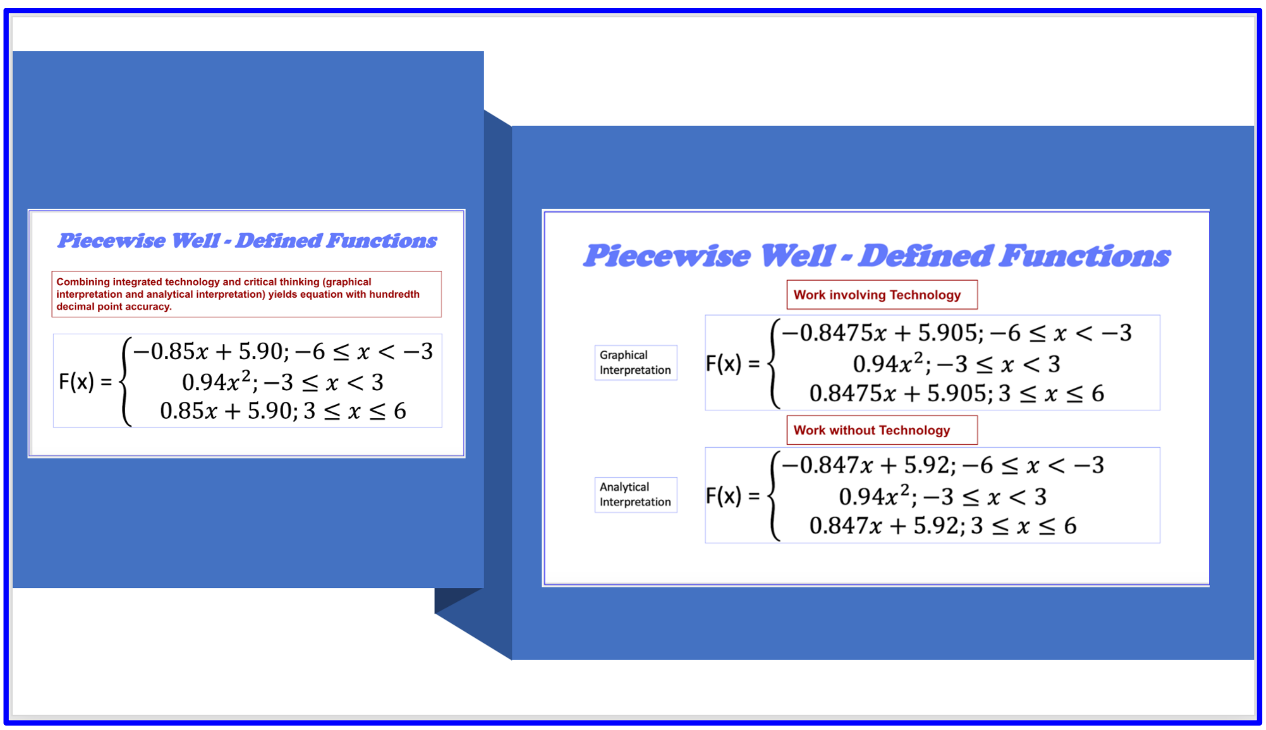

__Figure 3__. On the right window shows two equations of piecewise well defined functions: one equation comes as a result of using technology, and the other as a result of analytical work (work without technology). On the left side, combining integrated technology and critical thinking (graphical interpretation and analytical interpretation) yields equations with hundredth decimal point accuracy.

However, if we attempt to solve problems without taking into account all the pertinent parameters and conditions, the outcomes are likely to be erroneous, resulting in misinformation. Teachers who solve problems using markers on the board or through traditional means (teaching without technology) lack the ability to accurately depict geometric figures to scale and with proper proportions. Frequently, we illustrate figures in a manner that suits our requirements, yet this approach can sometimes lead to inaccuracies. Each interpretation remains partial, intrinsically tied to the interpreter's inherently contingent subjective standpoint (Žižek, 2012, p. 359).

It's important to recognize that relying solely on an analytical interpretation of a mathematical problem's solution doesn't necessarily encompass a comprehensive and thoughtful solution.

Integrating technology judiciously into instructional practices substantially enriches the content and significantly clarifies complex concepts. In Figure 2, the graphical interpretation facilitated by Desmos employs a trial-and-error approach to determine the optimal scaling factor, denoted as 'a = 0.94', which yields the most accurate quadratic equation representing the distinctive 'u-shape' of the insect. Employing trial-and-error, a solution is intuitively derived through experimentation to observe the outcomes (Bartlett, 2018).

Moreover, the linear functions effectively capture the length and direction of the insect's antennae. The subsequent step involves the utilization of technology to generate a comprehensive graph of the well-defined piecewise function, displayed in red. Desmos and Geogebra excel in this task, serving as outstanding tools. However, their effectiveness is amplified when guided by human input (Mastantuono, 2021).

The complementary section of the figure showcases the analytical approach (devoid of technology) used to derive equations for both the quadratic and linear functions within the limited domain.

The algebra project produces two distinct piecewise functions: one equation emerges from employing technology, while the other arises from analytical work, as depicted in Figure 3. In both equations, the determination of coefficients and constants for the linear equations involves calculations to several decimal places. A slight disparity between the linear equations generated through graphical interpretations and those derived analytically is observed. Nevertheless, the amalgamation of integrated technology and critical thinking (via graphical and analytical interpretations) culminates in equations accurate to the hundredth decimal place.

2.2. Reviewing Literature

As a general principle, the integration of technology in education aims to elevate the efficacy of teaching and learning to an advanced level. Often, teachers strive to impart optimal instruction in their sessions, while students grapple with a comprehensive understanding of mathematics. Alarmingly, approximately 79% of students identify mathematics as a significant hurdle in their learning journey (Pisa, 2015).

For educators to deliver lessons effectively, with lucid explanations and a coherent structure, it's imperative for them to blend technology into their teaching methods while exercising critical thinking judiciously. Conversely, students should also employ technology sensibly in their mathematics studies. Information and Communication Technology (ICT) serves as a valuable tool for meaningful learning when it offers students opportunities to learn with technology rather than simply from it (Jonassen et al., 2008).

The primary aim of mathematics teachers revolves around a dual focus: instructing without undue reliance on technology, and structuring content to encourage a higher level of critical thinking. Regardless of whether students possess average or challenging math proficiency, teachers should adapt lessons to require fewer steps in solving problems initially and progressively intensify the complexity of examples in line with students' progress. Technology, while supplementary, should function alongside teachers' efforts in the classroom. It supports and augments students' learning experiences while assisting teachers in refining their instructional approaches (Tramonti, Marinova, 2019).

Consequently, the astute combination of teaching methods involving both technology and non-technology approaches in mathematics substantially magnifies the impact of instructional practices within the classroom, creating a marked difference in learning outcomes.

The judicious application of technology, when employed optimally and in appropriate contexts, has a profound impact on the enhancement of mathematical thinking. Choi-Koh (1999) demonstrated how technology, specifically Geometer's Sketchpad (GSP), rapidly elevated a student's understanding of geometry from one level to the next. In the realm of geometry, Geometer's Sketchpad and its more advanced counterpart, Geogebra, accurately depict geometrical figures to scale and denote various dimensions of these shapes.

Moreover, students with a penchant for visual learning inherently gravitate towards translating problems into graphical representations as a means of solving them (Mianali, 2021). Once these concepts are transcribed into geometrical figures, they can be effectively inputted into digital tools. Groman (1996) uncovered that by utilizing GSP, students could construct medians of triangles and subsequently formulate conjectures, which in turn could progress to the development of proofs and higher-order thinking.

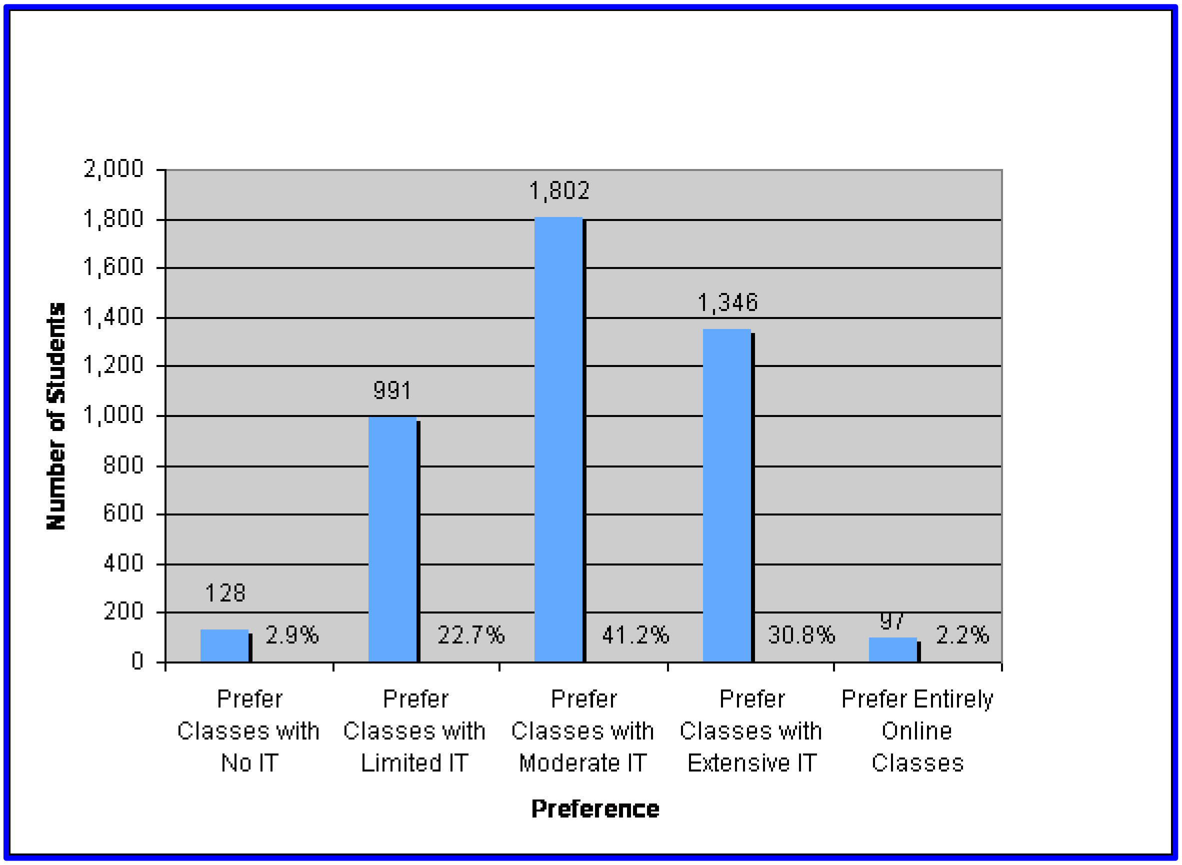

Figure 4. *Student Preference for Use of IT in Classes (N=4,363). The figure is the work of EDUCAUSE (Kvavik, n.d).*

Employing technology intelligently undoubtedly contributes to a substantial augmentation in the elevation of students' critical thinking abilities. Such thoughtful use of technology significantly raises the bar for students' cognitive engagement.

Throughout the paper, I have repeatedly used the term "teaching without technology," which I am referring to as analytical work or critical thinking. Irrespective of their proficiency in the subject, students often resort to a single strategy to navigate their academic journey: memorizing the material and hoping it aligns with exam questions (Halonen, 1996). Numerous courses necessitate preparation for multiple-choice exams featuring closed-ended answers. Consequently, students tend to bypass multifaceted analysis of questions through the lens of critical thinking.

Critical thinking is a concept defined in various ways, but I personally favor a straightforward interpretation. Critical thinking entails the capacity to objectively analyze information and arrive at reasoned judgments. It encompasses the assessment of sources, including data, facts, observable phenomena, and research findings (Doyle, 2022). Figure 5 outlines six essential components of critical thinking: Analysis, Inference, Reasoning, Generation, Interpretation, and Evaluation. Fundamental principles of robust critical thinking models involve recognizing challenges and formulating potential solutions (Black Sheep Community, 2022).

The essence of exceptional mathematical instruction hinges on robust critical thinking. Indeed, critical thinking serves as a robust foundation for effective teaching, analogous to how advancement to more complex math courses requires a mastery of fundamental mathematical concepts (Myers, 2009). Profound teaching devoid of excessive reliance on technology is inherently anchored in critical thinking – and it is through this mode of thinking that substantial learning flourishes.

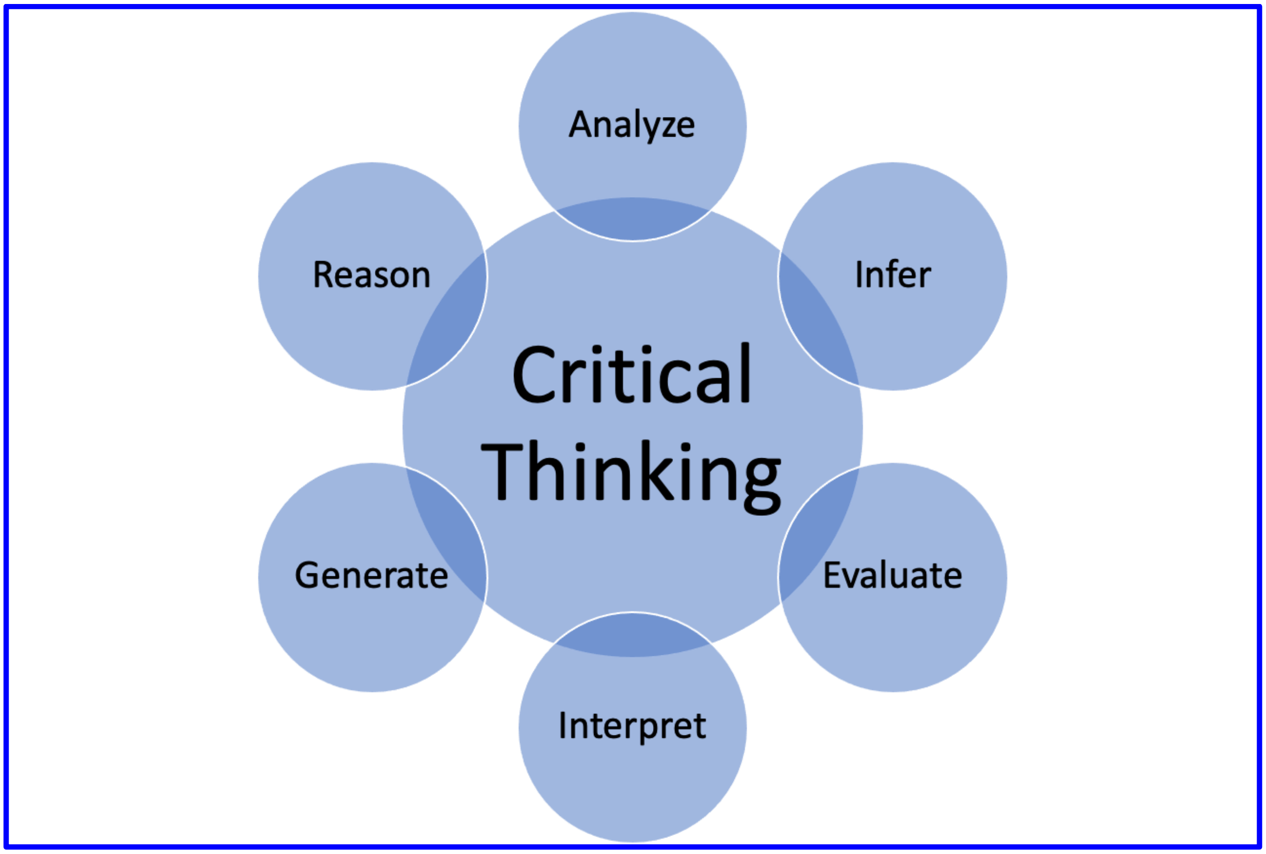

Figure 5. *The domain of critical thinking is very wide, but the figure narrows to the main six components: Analyze, Infer, Reason, Generate, Interpret, and Evaluate. The figure is modified based on the (Black Sheep Community, 2022).*

Critical thinking serves as a crucial tool for identifying patterns and examining real-world instances within any educational context, including mathematical lessons involving concepts and mathematical examples. Contemporary education places less emphasis on rote memorization. Instead, the focal point of the educational process lies not in the accumulation of information but in the comprehension of connections within and between the subjects being explored (Lipman, 2003). This educational philosophy harmonizes with a widely accepted definition of mathematics as the science of unveiling relationships amidst patterns.

Teachers should employ critical thinking within their instructional methods, aiming to discern connections among patterns. By doing so, students can learn through observing the lesson plan. Sternberg and Williams (2002) highlighted that students might not necessarily require formal instruction in critical thinking, as thinking inherently transpires within everyone as a natural process. This innate cognitive process encompasses the inclination to apply learning in novel contexts and the recognition that real-world predicaments are intricate and rarely possess a singular straightforward solution (Kennedy et al., 2010).

Frequently, when math teachers address real-world scenarios using a singular approach, they might subsequently discover a more straightforward method. Through diligence and experience, educators can hone their critical thinking abilities, enabling them to decipher patterns more efficiently and with reduced effort.

Combining rational approaches to teaching mathematics with and without technology yields seamless instructional experiences and significantly enhances students' learning outcomes. Several factors contribute to effective teaching, including the level of teachers' critical thinking abilities and their proficiency with digital tools. Additionally, a teacher's teaching philosophy holds considerable sway, influencing instructional methods by aligning them with the teacher's perception of students' preferred learning styles. Hence, the impact of teacher perceptions regarding student learning styles should not be underestimated (Choy, 2009).

Furthermore, teachers' proficiency with digital technology is a pivotal factor that can reshape and enhance instructional practices. When I assert that the strategic integration of teaching mathematics using both technological and non-technological methods serves as a powerful tool for optimal instruction, there are essentially classifying five distinct modes of teaching within this framework:

- Teaching only with technology
- Teaching with technology and limited without technology
- Combining rationally teaching with and without technology
- Teaching without technology and limited with technology
- Teaching without technology

My assertion is that the most effective teaching approach involves a rational combination of teaching mathematics with and without technology. This claim finds support in the visual data presented in Figure 4, which exhibits five analogous categories to the teaching classifications mentioned earlier.

In Figure 4, a total of 3,363 students were surveyed using five questions or preferences: a) preference for classes without IT, b) preference for classes with limited IT, c) preference for classes with moderate IT, d) preference for classes with extensive IT, and e) preference for entirely online classes. Notably, the preference for classes with moderate IT (choice - c) garnered the highest number of preferences. This preference aligns closely with the rational combination of teaching with and without technology, substantiating the effectiveness of this approach

3. Results

This research case study, rooted in inductive analytical research, yields several outcomes. Due to the paper's confined scope, only seven potential results are presented. The preeminent outcome underscores that the strategic fusion of teaching mathematics with and without technology bestows seamless instruction and significantly enhances students' learning outcomes to a considerable extent, thereby corroborating the principal theme of the paper. Another outcome of paramount significance within this study is the realization that the judicious combination of teaching with and without technology may give rise to fresh conjectures and, in turn, novel theorems.

- The mathematical model might not represent the real world exactly
- The presence of piecewise well defined functions (quadratic and linear functions) in the real world.
- Combining rationally teaching mathematics with and without technology imparts smooth instructions and elevates students' learning results to an extended degree.
- Analytical (teaching without technology) and graphical interpretations (Teaching with technology) yield similar results.

- Mathematical concepts that we learn in high school are applicable in the real world
- Combining teaching with and without technology rationally might create new conjectures, respectively new theorems.
- The literature confirms the students prefer classes with moderate IT to other four different options: a) prefer classes with no IT, b) prefer classes with limited IT, d) prefer classes with extensive IT, and e) prefer entirely online classes.

The secondary findings of this research hold immense significance as they lend a logical basis for the practical application of high school mathematics, particularly algebra, in real-world contexts. The project serves as a tangible demonstration of how piecewise functions can be effectively employed in real-world scenarios. However, it's imperative to leverage critical thinking to bridge the gap between real-world geometrical figures and mathematical concepts, ensuring an approximation that maintains a reasonable level of accuracy.

4. Discussion

Typically, quadratic equations (and functions) constitute an integral component of high school curricula across the globe. Almost every national standardized test mandates that students tackle quadratic equation problems (Kim et al., 2021). Within the realm of quadratic equations, two predominant forms emerge: the vertex form and standard form. An additional, albeit less prevalent, format is the factored form. Notably, the factored form of quadratic equations offers distinct advantages over the aforementioned two forms, especially concerning the identification of critical points (x-intercepts).

The quadratic function (or equation) in standard form provides an advantage when seeking the critical point, specifically the y-intercept. Conversely, the vertex form of a quadratic function (or equation) holds an edge over the other two formats when it comes to determining the parabola's vertex. Each variation of the quadratic function boasts its own set of advantages and drawbacks. Researchers and educators have meticulously explored these diverse forms of quadratic functions, recognizing their role in establishing a robust mathematical groundwork for high school students.

Find the vertex of the quadratic functions in factored form

a) $y = (x+2)(x-4)$ b) $y = (x+1)(x-7)$ c) $y = -3(x-2)(x+6)$

a) $y = (x+2)(x-4)$

$x_1 = -2, \quad x_2 = 4$

$x = \frac{x_1+x_2}{2} = \frac{-2+4}{2} = 1 = h$

$k = f(x) = f(1)$

$= (1+2)(1-4)$

$= (3)(-3)$

$\mathbf{k = -9}$

$\boldsymbol{Vertex} - \boldsymbol{V}(\mathbf{1}, \mathbf{-9})$

b) $y = (x+1)(x-7)$

$x_1 = -1, \quad x_2 = 7$

$x = \frac{x_1+x_2}{2} = \frac{-1+7}{2} = 3 = h$

$k = f(x) = f(3)$

$= (3+1)(3-7)$

$= (4)(-4)$

$\mathbf{k = -16}$

$\boldsymbol{Vertex} - \boldsymbol{V}(\mathbf{3}, \mathbf{-16})$

c) $y = -3(x-2)(x+6)$

$x_1 = 2, \quad x_2 = -6$

$x = \frac{x_1+x_2}{2} = \frac{2-6}{2} = -2 = h$

$k = f(x) = f(-2)$

$= -3(-2-2)(-2+6)$

$= -3(-4)(+4)$

$\mathbf{k = 48}$

$\boldsymbol{Vertex} - \boldsymbol{V}(\mathbf{-2}, \mathbf{48})$

Conjecture: Analytical process on finding k yields k = (m)(-m) when $m \in \mathbb{R}$.

Figure 6. *Examples of quadratic functions/equations in factored form. Finding the vertex, respectively, the coordinate k implies at a certain point* $k = (m)(-m)$ *when* $m \in \Re$.

Exploring quadratic equations, both with and without the aid of technology, endeavors to foster more conducive conditions for students to comprehend the various approaches and techniques for solving quadratic equations. Broadly speaking, quadratic equations present challenges to many students, manifesting in diverse forms like struggles with algebraic procedures—particularly when factoring quadratic equations—and difficulties in attributing significance to these quadratic expressions (Didis, Erbas, 2015).

By employing rational instructional methods coupled with critical thinking and the judicious application of technology, students can achieve a deeper grasp of concepts and mathematical strategies required to solve quadratic equations in any of their three forms. Furthermore, through consistent practice, thorough elaboration, and the exploration of quadratic functions from varied angles, the learning outcomes are significantly enriched.

Quadratic functions are a staple of the curriculum in high schools, encompassing all three forms: standard form, vertex form, and factored form. Within the lesson plans, each of these forms is covered for a minimum of one week, comprising various topics such as quadratic function introduction, progressive problem-solving from simple to complex instances, conversion between different forms of quadratic functions, and practical applications of quadratics in real-world scenarios. The process of teaching and learning quadratic functions necessitates a substantial time investment. To illustrate, the conversion of quadratic functions from standard form to vertex form alone requires a minimum of five examples, equivalent to one instructional session. This conversion process repeats for transitioning between vertex form, standard form, and factored form, and vice versa. Proficiency in these lessons demands a significant duration of time dedicated to mastery.

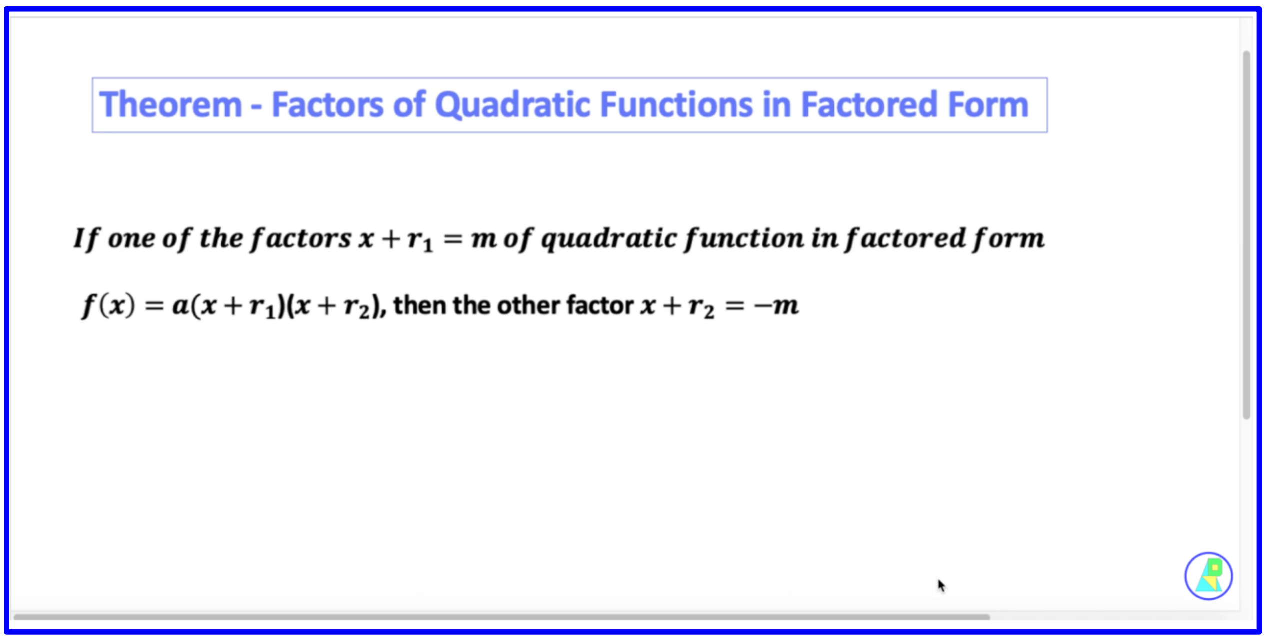

Figure 7. *The simple theorem on the factors of quadratic functions in the factored form. The figure is taken from youtube channel Duli Pllana. Click the button to watch the proof* ⇒ Theorem - Factors of Quadratic Function in Factored Form

While I was engaged in solving quadratic equations in factored form during my class, using the examples presented in Figure 6, I stumbled upon a recurring pattern. This pattern emerges consistently in every instance when we determine the coordinate "k" of the vertex. The three examples illustrated in Figure 6 indicate that the value of "k" is the product of certain integers with opposite signs: k = (3)(-3), k = (4)(-4), and k = (4)(-4). At this juncture, I posed a critical thinking question to my students: Does this pattern repeat across all cases, and if so, what might be the underlying reason?

The question piqued the students' curiosity, prompting them to contemplate the occurrence of this specific pattern in the solution of mathematical problems. Engaging in critical thinking laid the foundation for potentially formulating a conjecture. Before proceeding, let me offer a concise definition of a conjecture: a statement that holds potential truth, often supported by reasoning or research, but remains unproven (Math is Fun, n. d.).

We collectively discussed the concept of a conjecture in the classroom and deliberated whether we could formulate a conjecture in this particular case. However, before making any claims, we acknowledged the necessity for further evidence, a process intricately tied to probability (Achinstein, 2001). Subsequently, we turned to technology to gain a more comprehensive perspective. Employing SYMBOLAB (a math solver and graphing calculator), we selected numerous examples in the factored form and executed calculations swiftly. As anticipated, all the instances exhibited the identical pattern in the "k" coordinate of the vertex. Notably, the "k" value in each case manifested as a product of integers sharing the same absolute value but with opposing signs.

Through the amalgamation of manual analysis and technological assistance during my instructional sessions, a novel conjecture emerged.

As technology affords us the capability to solve multiple examples swiftly and with precision, I proceeded to examine over fifty instances. Remarkably, all these examples exhibited the same recurring pattern, prompting me to contemplate the potential emergence of a theorem from this scenario. Allow me to offer a straightforward definition of a theorem before delving further: a theorem is a statement that can be logically proven as true using accepted mathematical operations and arguments (Wolfram, n.d).

After allowing technology to take a back seat, I embarked on an analytical exploration to determine whether this pattern could indeed lead to a possible theorem. Given that quadratic equations fall within the domain of elementary mathematics and the patterns at hand are readily discernible and uncomplicated, I commenced by scrutinizing the matter from a basic perspective. This involved observing the general form of the midpoint formula and substituting quadratic equations in their factored form. By applying the general midpoint formula to the general form of the factored quadratic equation, I unraveled the explanation for the recurrence of this pattern in the y-coordinate (k) of the vertex.

Ultimately, I formulated the theorem, conceptualized as a "homemade theorem," through a blend of technology and my YouTube channel, as depicted in Figure 7. Once again, the strategic fusion of both manual analytical work and technological assistance yielded a noteworthy and straightforward theorem.

5. Conclusion

The research case study focuses on investigating the impact of combining mathematics teaching with and without technology during instructional sessions and its subsequent effect on learning outcomes. While all high school math teachers equipped with smart boards or other advanced digital platforms incorporate technology into their mathematics lessons, not all of them adeptly blend technology with analytical thinking strategies in a rational manner. Among these educators, a spectrum emerges: some excessively rely on technology, some underutilize it, and only a few effectively merge technology with critical thinking in a judicious manner. The divergence in teaching approaches stems from varying experiences, levels of education, and individual teaching philosophies, highlighting the inherent diversity among educators. It's important to recognize that educational practices should be pragmatic and suited to the current context (Creswell, 2009).

It's worth noting that this paper does not address other significant teaching factors that also influence mathematical instruction. Its focus is confined to three primary components of mathematical instruction: the integration of mathematics teaching with and without technology, the exclusive use of mathematical teaching without technology, and the sole use of mathematical teaching with technology. Furthermore, there exist teachers who incorporate both technological and non-technological approaches to some extent but may not do so rationally. In such cases, the impact of such teaching methods might not yield an exceptional learning outcome.

Above all, the paper delves into the ample evidence validating the central theme of the study: that the strategic combination of teaching mathematics with and without technology not only facilitates seamless instruction but also significantly enhances students' learning outcomes.

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